PRIMJENA PLANIRANOG EKSPERIMENTA ZA ODREĐIVANJE FAKTORA INTENZITETA NAPONA K_I NA DUBOKIM DANCIMA POMOĆU FEM ANALIZA

APPLICATION OF THE PLANNED EXPERIMENT FOR DETERMINING THE STRESS INTENSITY FACTOR IN VESSEL HEADS USING FEM ANALYSIS

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REZIME
Ovaj rad se bavi numeričkim određivanjem faktora intenziteta napona K_I za meridijalne pukotine na dubokom dancetu posuda pod pritiskom. Eksperiment je proveden prema odabranim parametrima, uključujući prečnik posude, debljinu stijenke i pritisak. Analizom rezultata zaključeno je da su cilindrični segment dubokog danceta i kruna danceta najkritičnija mjesta za pojavu pukotina, izložena zateznim naprezanjima. Regresionom analizom utvrđeno je da unutrašnji pritisak ima najveći utjecaj na faktor K_I, dok je za krunu dubokog danceta značajan i prečnik posude. Provedeni eksperimenti potvrđuju dobijene regresione modele, iako su primijećena odstupanja koja se pripisuju nesavršenostima numeričkih simulacija. Unatoč tome, zaključuje se da su dobijene vrijednosti faktora K_I zadovoljavajuće tačne.

Professional paper

SUMMARY
This paper deals with the numerical determination of the stress intensity factor K_I for meridional cracks on the ellipsoidal head of pressure vessels. The experiment was carried out according to selected parameters, including vessel diameter, wall thickness and pressure. The analysis of the results concluded that the cylindrical segment of the ellipsoidal head and the crown of the ellipsoidal head are the most critical places for the appearance of cracks, exposed to tensile stresses. Regression analysis determined that the internal pressure has the greatest influence on the K_I factor, while the diameter of the vessel is also significant for the crown of the ellipsoidal head. The conducted experiments confirm the obtained regression models, although deviations attributed to the imperfections of the numerical simulations were observed. Despite this, it is concluded that the obtained values of the K_I factor are satisfactorily accurate.

1. INTRODUCTION

The task of this paper is to numerically determine the values of the stress intensity factor K_I for meridional cracks on the ellipsoidal head of pressure vessels, according to the selected diameter of the vessel, wall thickness and pressure. It is necessary to conduct a planned experiment and on the basis of its results to make conclusions about the most influential input factors affecting the value of this stress intensity factor. Also, the results will serve to confirm whether the obtained regression equations of the value of the stress intensity factor correspond to arbitrarily chosen values of the input factors that fall within the limit of those factors. The geometry of the pressure vessel ellipsoidal head is created according to the DIN 28013 [1] ellipsoidal head standard, as shown in the following figure. The length of the cylindrical part of the ellipsoidal head is 175 mm, and it was selected through a detailed analysis in order to obtain the highest value of the stress intensity factor K_I on each vessel at each crack size without the influence of other pressure vessel segments on the crack. Among the other geometric characteristics of the vessels, it is necessary to distinguish the height of the coat, which is adopted to be 3 diameters of the vessel.

The chosen shape of the cracks is the one most frequently encountered in practice, i.e. semi-elliptical [4].

2. THE SETTING OF THE PLANNED EXPERIMENT

It was chosen that a full experiment should be conducted with 3 input factors at 2 levels, with 2 replicates at the zero point, giving a total of N = $2^3 + 2 = 10$ experimental points [3]. There should be 2 output parameters and they are the values of the stress intensity factor K_I for the most critical places for a crack on the ellipsoidal head. The constant parameters of the experiment are:

- the crack is in the meridional direction and from the outer surface of the ellipsoidal head;
- the material of the pressure vessel is steel P355 with yield stress $R_{eH} = 355$ MPa;
- the depth of the crack is equal to half the thickness of the wall of the ellipsoidal head and
- dimensional characteristics of the crack a=c.

The input factors of the experiment are the internal pressure of the vessel Pi, the diameter D, and the wall thickness of the vessel under pressure t.

Table 1. Levels of input factors used in the planned experiment

Level	Factors		
	$\mathbf{P_i}$, MPa	**D**, mm	**t**, mm
(+1)	1,5	2500	14
(-1)	0,5	1250	6
(0)	1,0	1875	10

Table 2. Plan of matrices of the experiment

Case	X_1	P_i	X_2	D	X_3	T
1, 2	0	10	0	0,6	0	10
3, 4	0	10	0	0,6	0	10
5	-1	5	-1	0,2	-1	6
6	-1	5	-1	0,2	1	14
7	-1	5	1	1	-1	6
8	-1	5	1	1	1	14
9	1	15	-1	0,2	-1	6
10	1	15	-1	0,2	1	14
11	1	15	1	1	-1	6
12	1	15	1	1	1	14

As can be seen from the table, the experiment plan is fully orthogonal with one block where all the experiment points are.

A better representation of the experimental points with the input factors coded is given in the following figure, where it can be seen that the experimental points are actually the endpoints of this closed box, with the central point as a control. After conducting an experiment for these experimental points and obtaining regression models, the resulting models should satisfy the values for any point that falls within this closed box.

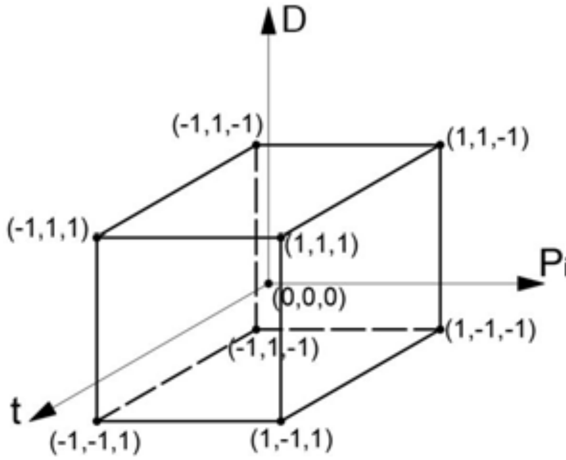

Figure 1. *Locations of experimental points with coded input factors*

3. DETERMINING THE OUTPUT PARAMETERS OF THE EXPERIMENT

The output parameters of the planned experiment are the values of the stress intensity factor K_I for the 2 most critical places for a crack on the ellipsoidal head, so the experiment will have 2 output parameters [5]. In order to determine the most unsuitable places for a crack, it is necessary to place the crack in several places on the ellipsoidal head, according to the highest component stress for certain segments of the ellipsoidal head [4]. Given that these are meridional cracks, for the K_I factor it is necessary to observe the circular component stresses for certain segments of the ellipsoidal head. It is now necessary to first determine the validity of the numerical simulations, that is, to compare the values of the component stress obtained numerically with the expected values obtained analytically. This was done at the zero point of the experiment, i.e. on the container wall t = 10 mm and with internal pressure $P_i = 10$ bar. The spherical segment of the ellipsoidal head, i.e. the crown, as the name suggests, has the shape of a sphere, so the formulas for the meridional and circular component stress of a spherical vessel under pressure can be applied to it, with these calculated values valid for the top of the ellipsoidal head crown itself:

- Meridial component stress

$$\sigma_1 = \frac{P_i \cdot R_1}{2 \cdot t} = \frac{1 \ \cdot 0{,}8 \cdot 1875}{2 \cdot 10} = 75 \ MPa \quad (1)$$

- Circular component stress

$$\sigma_2 = \sigma_1 = 75 \ MPa. \quad (2)$$

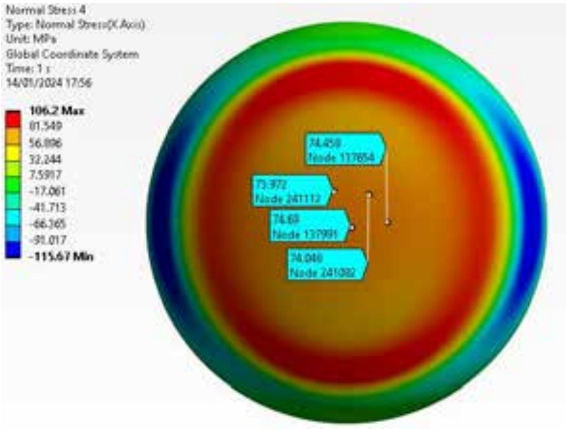

Figure 2. *Meridional stress component on the spherical segment of the ellipsoidal head*

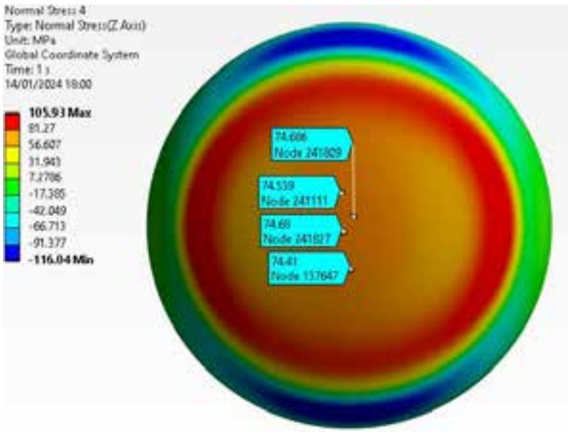

Figure 3. *Circular stress component on the spherical segment of the ellipsoidal head*

Since the height of the cylindrical segment of the ellipsoidal head is too small, the proximity of the junctions with the other segment of the ellipsoidal head, as well as with the vessel shell, would have no effect on its component stresses. The meridional and circular component stresses were checked for the pressure vessel shell, because it has the same shape as the cylindrical segment of the ellipsoidal head, but without the influence of other elements of the vessel.

- Meridial component stress

$$\sigma_1 = \frac{P_i \cdot D}{4 \cdot t} = \frac{1 \ \cdot 1875}{4 \cdot 10} = 46{,}87 \ MPa \quad (3)$$

- Circular component stress

$$\sigma_2 = \frac{P_i \cdot D}{2 \cdot t} = \frac{1 \ \cdot 1875}{2 \cdot 10} = 93{,}75 \ MPa. \quad (4)$$

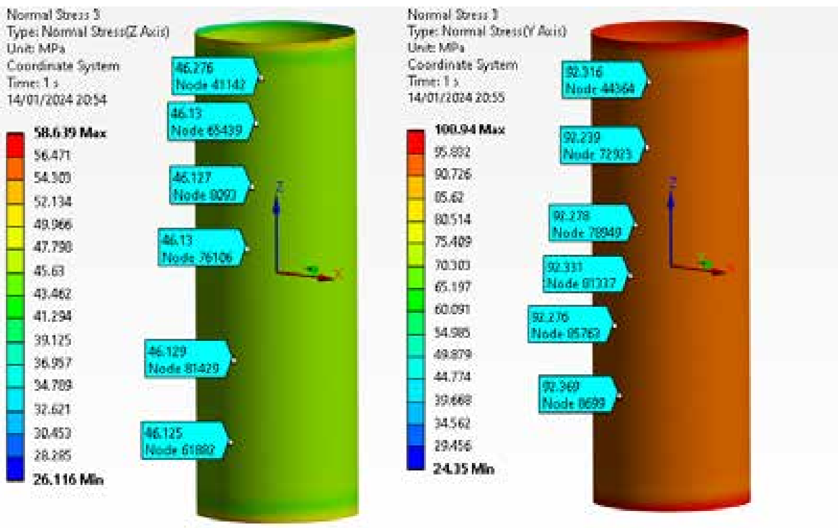

Figure 4. *Stress component on the pressure vessel shell – meridional (left) and circular (right)*

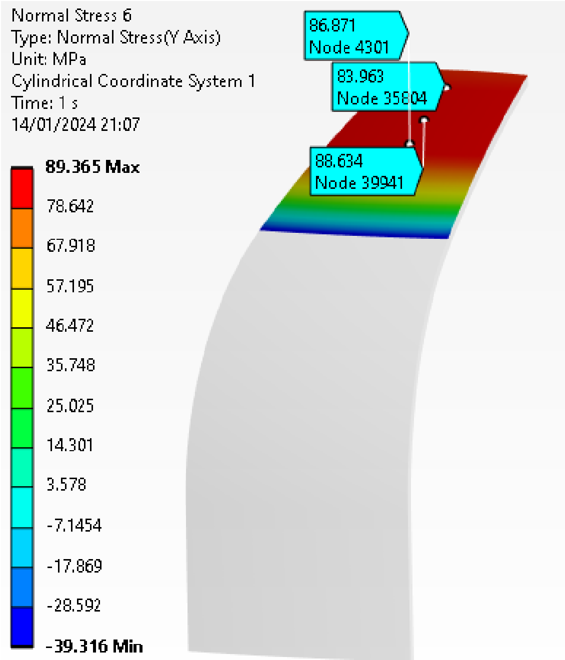

Figure 5. *Maximum circular stress component on the ellipsoidal head crown*

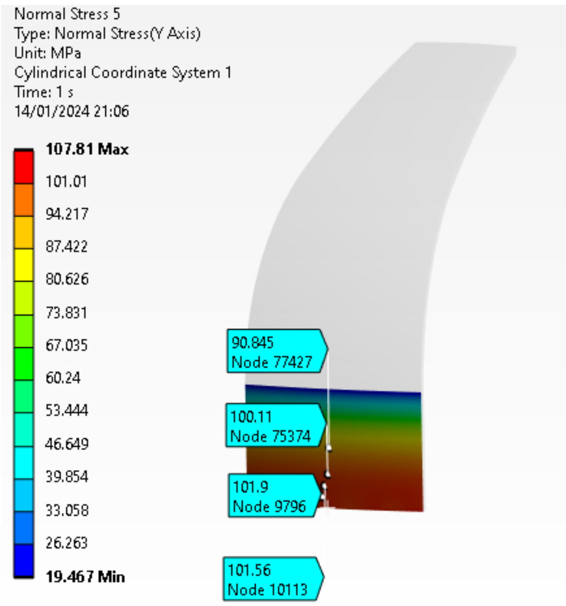

Figure 6. *Maximum circular stress component on the ellipsoidal head cylinder*

Circular stresses are important here due to their direction of action because they spread the meridional crack, and the goal is to obtain the highest values of the K_I factor, i.e. the most critical places for the appearance of meridional cracks on the ellipsoidal head.

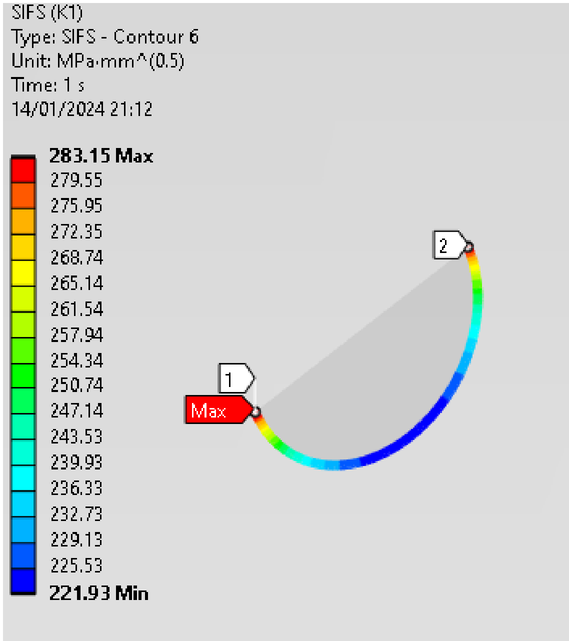

Figure 7. *K_I factor value on the ellipsoidal head crown*

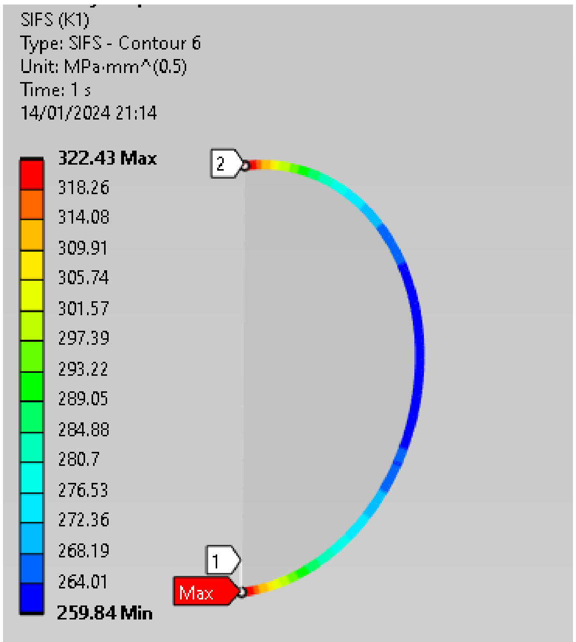

Figure 8. *K_I factor value on the ellipsoidal head cylinder*

In the following figure, there are the values of the stress intensity factor K_I for all placed cracks. The highest values occurred precisely at the points of action of the highest circular component stresses on the crown and the cylindrical segment of the ellipsoidal head, as it was assumed. Therefore, these 2 cracks, i.e. the values of the K_I factor for cracks at these 2 places, were taken as the output parameters of the experiment.

4. PROCESSING OF THE EXPERIMENT RESULTS

After determining the locations of the most critical cracks and certain values of the stress intensity factor K_I for cracks on the crown and the cylindrical segment of the ellipsoidal head at the zero point of the experiment, it is necessary to carry out numerical simulations to obtain these factors and for all other points of the experiment. The cracks for all points of the experiment were placed in mostly the same places as at the previously examined zero point of the experiment. Using software for statistical data processing, it is necessary to address the obtained results.

Table 3. *Results of the conducted planned experiment*

Case	X_1	P_i, bar	X_2	D, mm	X_3	t, mm	$K_{i,crown,}$MPa√m	$K_{i,cylin,}$MPa√m
9	-1	5	-1	1250	1	14	6,73	7,92
5	1	15	-1	1250	-1	6	11,31	12,18
10	-1	5	1	2500	-1	6	6,74	8,29
6	1	15	1	2500	1	14	15,2	18,29
1, 2	0	10	0	1875	0	10	8,95	10,19
11	-1	5	-1	1250	-1	6	3,82	4,11
7	1	15	-1	1250	1	14	8,11	9,41
12	-1	5	1	2500	1	14	5,04	6,06
8	1	15	1	2500	-1	6	17,81	25,82
3, 4	0	10	0	1875	0	10	8,95	10,19

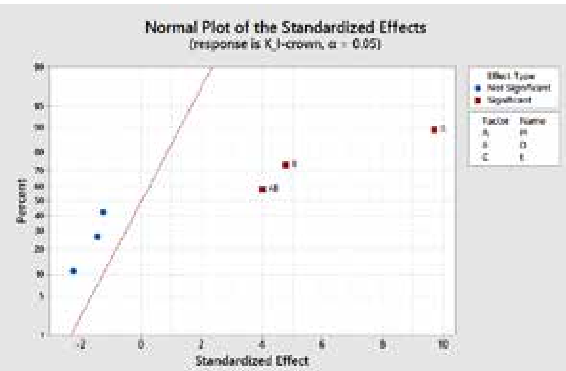

Figure 9. *Significance of extended model factors for $K_{I\text{-}crown}$ output parameter*

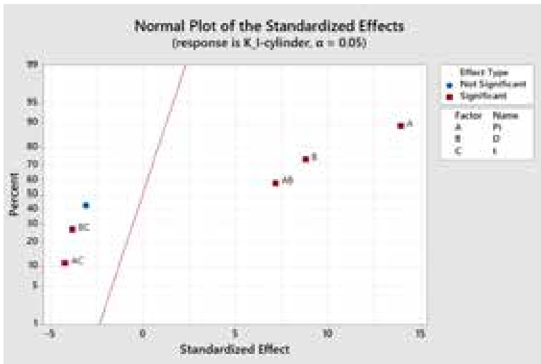

Figure 10. *Significance of extended model factors for $K_{I\text{-}cylinder}$ output parameter)*

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.998773	99.18%	97.55%	89.99%

Figure 11. *Correlation coefficient of the $K_{I\text{-}crown}$ factor model for the extended model*

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.09995	97.92%	93.75%	12.38%

Figure 12. *Significance of extended model factors for $K_{I\text{-}cylinder}$ output parameter)*

The regression equation for the $K_{I\text{-crown}}$ factor for the extended model is:

$$K_{I-crown} = 9{,}27 + 3{,}76X_1 + 1{,}85X_2 + 1{,}55X_1X_2. \quad (5)$$

The regression equation for the $K_{I\text{-cylinder}}$ factor for the extended model is:

$$K_{I-cylinder} = 11{,}25 + 4{,}92X_1 - 3{,}11X_2 + 2{,}52X_1X_2 - 1{,}35X_2X_3 - 1{,}48X_1X_3 \quad (6)$$

5. VERIFICATION OF THE OBTAINED REGRESSION MODELS WITH ARBITRARY FACTORS

After processing the data obtained by conducting the experiment and obtaining a regression model for the output parameters of the experiment, it is necessary to check the obtained models at an arbitrary point that falls within the closed box of the experiment, i.e. within its boundaries. The following factor values in coded coordinates were selected for verification: X_1 = 0,5; X_2 = –0,60 and X_3 = 0,50. According to the previously obtained regression equations, the following values of the output parameters are expected for these arbitrarily chosen factors:

$$K_{I-crown} = 9{,}56\ MPa\sqrt{m}$$

$$K_{I-cylinder} = 11{,}11MPa\sqrt{m}.$$

To carry out numerical simulations, it is necessary to translate these coded factors into actual numerical values of the input factors of the experiment, as follows:

-internal pressure:

$$X_1 = 0{,}50 \quad \Rightarrow \quad P_i = 10 + (15 - 10) \cdot 0{,}5 = 12{,}5\ bar \quad (7)$$

-vessel diameter:

$$X_2 = -0{,}60 \quad \Rightarrow \quad D = 1875 - (1250 - 1875) \cdot (-0{,}60) = 1500 \quad (8)$$

- the thickness of the vessel wall:

$$X_3 = 0{,}50 \quad \Rightarrow \quad t = 10 + (14 - 10) \cdot 0{,}50 = 12\ mm \quad (9)$$

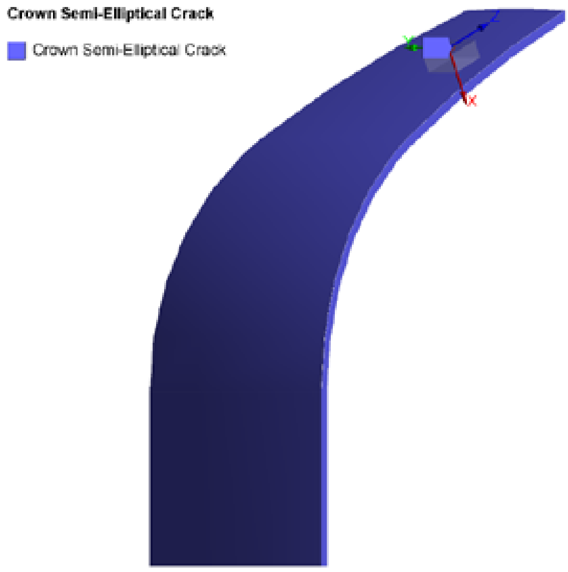

Figure 13. *The location of the placed crack on the crown*

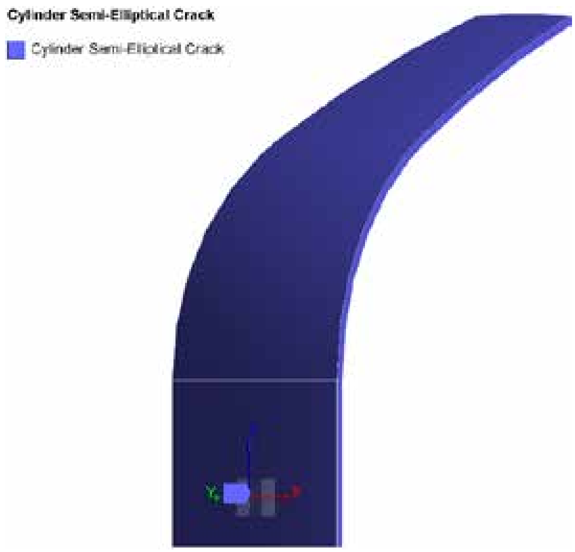

Figure 14. *The location of the placed crack on the cylinder*

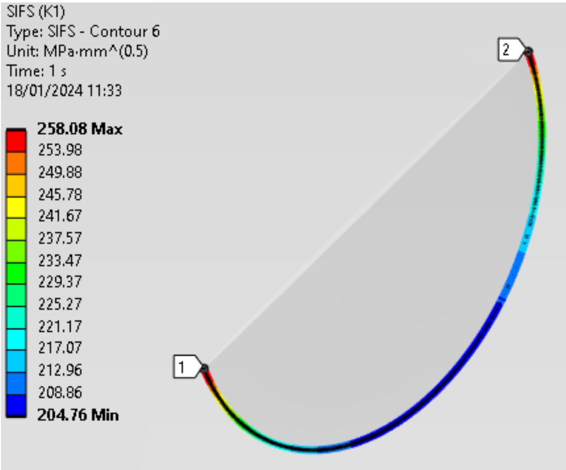

Figure 15. *$K_{I\text{-}crown}$ factor values for an arbitrary experimental point*

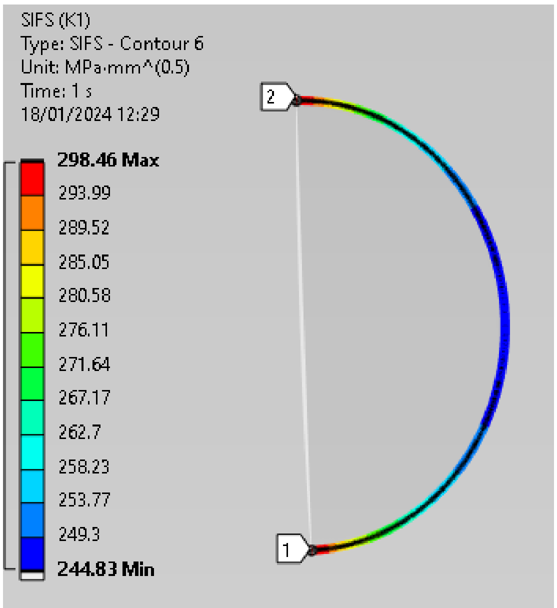

Figure 16. *$K_{I\text{-}cylinder}$ factor values for an arbitrary experimental point*

Table 4. *Results of the planned experiment*

	Expected value [MPa·m$^{1/2}$]	Numerical value [MPa·m$^{1/2}$]	Deviation %
$K_{I\text{-}crown}$	9,56	8,22	-14,22
$K_{I\text{-}cylinder}$	11,11	9,44	-15,03

As can be seen from the table, the numerically obtained values of the output parameters of the K_I factor for an arbitrary point do not completely match the expected values according to the regression models. The deviations are 14,22% and 15,03% for both parameters and are negative in nature, i.e. numerically obtained values are in both cases equally less than the expected values obtained by regression. This completes the verification of the obtained regression models, and it is necessary to determine the reasons for these deviations, and to draw certain final conclusions.

6. CONCLUSION

In this work, it was concluded that the cylindrical segment of the ellipsoidal head and the crown of the ellipsoidal head under pressure vessel, or more precisely the places of action of the largest circular component stresses on those segments of the ellipsoidal head, are the most critical places for the appearance of meridional cracks. What is important is that those places of action of the largest circular component stresses are exposed to tensile stresses and not to compressive stresses, because tensile stresses open the crack and thus have an adverse effect on the integrity of the pressure vessel, while compressive stresses help to close the crack.

By conducting the planned experiment in full, it was concluded that, under the given conditions of the experiment and at the selected input factors, higher values of the stress intensity factor K_I occur on the cylindrical segment of the ellipsoidal head than on the crown of the ellipsoidal head, and at all experimental points and at the arbitrary experimental point, which finally checked the obtained regression models. It can therefore be concluded that the cylindrical segment of the ellipsoidal head is less suitable for the appearance of cracks, and the reason for that is a slightly higher circular component stress which occurs on that segment than on the crown of the ellipsoidal head, as shown in the pictures in the paper.

From the regression equations of the output parameters of the experiment, obtained by the extended statistical analysis of the conducted experiment data, it can be seen the greatest influence among selected input factors on the output value has the internal pressure P_i, because the coefficient next to that term in both regression equations is the largest, meaning that by increasing the internal pressure also increases the value of the K_I factor. In the case of the crown, from the obtained regression equation we see that the influence on the increase of the K_I factor, in addition to the internal pressure P_i, also has the diameter D, as well as their mutual relationship. When it comes to the regression equation of the K_I factor, its increase is influenced, as in the previous case, by the diameter D. With the fact that in front of the product of the internal pressure P_i and the diameter D, there is a + sign, which means that their product affects the increase of the K_I factor, while in front of the product internal pressure P_i and wall thickness t, and diameter D and wall t, there is a sign - which means that these two mutual relationships affect the reduction of the K_I factor.

By checking the obtained regression models at an arbitrary experimental point, certain deviations were obtained in relation to the expected values. Here it is concluded there is a possible reason for the appearance of such deviations, which is the imperfection of numerical simulations, which is always present in them. Directly related to this possible error is the network itself, which should be even smaller in order to increase the accuracy of these numerical simulations. It can be concluded that the values of the K_I factor obtained during the verification of the regression models are satisfactorily accurate.

7. REFERENCES

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