UTJECAJ PARAMETARA REZANJA NA PROCES OBRADE CNC REZAČEM STIROPORA KORIŠTENJEM EKSPERIMENTALNO-MATEMATIČKOG MODELIRANJA

INFLUENCE OF CUTTING PARAMETERS ON THE PROCESS OF STYROFOAM CNC CUTTER OPERATION USING EXPERIMENTAL-MATHEMATICAL MODELING

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REZIME

Pregled dosadašnjih istraživanja pruža temeljno razumijevanje trenutnog stanja u području procesa rezanja stiropora na CNC rezaču, identifikujući nedostatke i potrebe za daljnjim istraživanjem. Identifikacija važnih ulaznih parametara, poput brzine kretanja žice, gustoće stiropora i jačine struje, daje osnovu za planiranje eksperimenta. U ovom radu metodologija istraživanja je detaljno opisana, uključujući odabir faktora, nivoa, te plan eksperimentalnog dizajna (Box-Behnkenov dizajn). Analiza podataka dobijenih eksperimentom fokusirana je na izlazne veličine y_1 *(vanjska mjera) i* y_2 *(unutrašnja mjera), sa naglaskom na statističku analizu uz upotrebu softvera* Minitab *i* Microsoft Excel. *Kroz sveobuhvatan pregled, rad teži doprinijeti razumijevanju procesa rezanja stiropora na CNC rezaču, pružajući uvid u utjecaj parametara na kvalitet rezanja.*

SUMMARY

An overview of previous research provides a fundamental understanding of the current state of the process of cutting styrofoam on a CNC cutter, identifying gaps and the need for further research. Identification of important parameters, such as wire cutting speed, density of the styrofoam and current intensity, provide the basis for planning the experiment. The research methodology is described in detail, including the selection of factors, levels, and design of experiment (Box-Behnken design). The analysis of the data obtained by the experiment is focused on the output sizes y_1 *(external measure) and* y_2 *(internal measure), with an emphasis on statistical analysis using Minitab and Microsoft Excel software. Through a comprehensive review, the paper aims to contribute to the understanding and optimization of the process of cutting styrofoam on a CNC cutter, providing insights into the influence of parameters on the quality of cutting.*

1. INTRODUCTION

The process of cutting styrofoam on a CNC hot-wire cutter serves as an example and illustrates, via experiment, how to optimize significant parameters, while simultaneously defining a mathematical model that clarifies the influence of these parameters on the measurement results. By reviewing previous research, it can be concluded that until now the analysis of the influence of parameters on the given process was carried out with the search for the optimal level for each parameter separately, while in this work, both the mathematical model and the influence of individual parameters on the accuracy of external and internal measurements, separately, are defined. Properties of styrofoam as a material, such as wire cutting speed, styrofoam density and current intensity, are the input parameters for the experimental design. The hypothesis assumed that the fundamental three parameters would exert the most significant influence on the experimental outcome. This research details the precise methodology, which includes the selection of factors, levels, and the implementation of the Box-Behnken design for the experiment. The focus of data analysis is on the output variables y_1 (external measure) and y_2 (internal measure), with special emphasis on statistical analysis and validation of results using software such as

Minitab and Microsoft Excel. In the chapter dedicated to the presentation and interpretation of the experimental results, a comprehensive analysis of the results for both output variables was done, followed by statistical and graphical interpretations. The goal of this research is to establish a mathematical model that describes the influence of all parameters on the accuracy of the measurement results, and to determine the most optimal combination of cutting parameters to obtain the smallest possible deviations of the dimensions of the cut pieces of styrofoam compared to the ideal geometry.

1.1. The operating principle of CNC styrofoam hot wire cutter

The operating principle of CNC styrofoam cutter with a hot wire is essentially simple: electrical energy passes through the wire, heating it to the desired temperature according to the relationship between input voltage and electrical resistance. When the wire touches the foam surface, it melts the surrounding area, the heat from the wire vaporizes, and a smooth surface forms on the foam. The material being cut softens below its melting point, because it is a thermoplastic material, and after that the cutting follows. The speed change and wire diameter affect the surface and width of the cut. A CNC styrofoam cutter with a hot wire is a sophisticated system that enables precise cutting of styrofoam using controlled heat. It consists of several key components. The styrofoam board is supported by spikes, ensuring a stable and firm position of the input styrofoam material for further processing. There is also a wire used for cutting the styrofoam, and a mechanism driven by motors to move the wire along two axes (2D) to achieve desired shapes and dimensions. Additionally, the system includes a control unit that manages all aspects of cutting, as well as software for programming and controlling the cutting process. Through programmed commands from the control unit, by adjusting the electrical current strength, the wire is heated to a specific temperature and precisely moves along the x and y axis to cut the desired shape or pattern in the styrofoam. The wire movement speed can be controlled to achieve the desired cutting results.

Figure 1 *CNC hot wire cutter*

2. EXPERIMENTAL SETUP

The problem, or experiment intended to be conducted here, is based on initially cutting pieces of styrofoam on a CNC styrofoam cutter, of a predetermined shape. Then, the external and internal dimensions of the cut piece of styrofoam will be measured. In this experimental procedure, the same shapes of styrofoam will be cut for three different types of styrofoam (different densities) and with different cutting parameters (current intensity, wire cutting speed). The experiment aims to determine a mathematical model that would describe the influence of the mentioned parameters, as well as to determine the optimal combination of cutting parameters to obtain minimal deviations of the dimensions of the cut pieces of styrofoam compared to their ideal geometry (CAD model). Specifically for conducting the experiment of cutting styrofoam, the following equipment is required: a CNC styrofoam cutter with hot wire and its corresponding components, styrofoam material, and a vernier caliper.

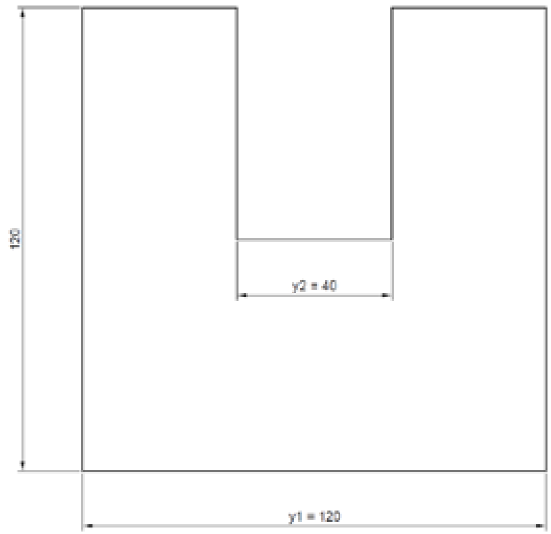

Figure 2 *Dimensional characteristics of the observed model*

2.1. A šlan matrix and selection of experimental model

The step preceding the process of conducting the experiment is the selection of experimental model according to which the experiment will be executed and the results mathematically processed. The experiment involves varying three factors:

- styrofoam density (ρ),
- wire cutting speed (v),
- current intensity (I).

Other influential parameters such as wire thickness and wire material are kept constant. In defining the shape function $y = f(\rho, I, v)$, the Box-Behnken model is used. Table 1 shows the selected natural and corresponding coded values of the factors.

Table 1 Natural and coded values of the model factors

FACTORS		Low level	Central point	High level	Variation
Density	ρ $[kg/m^3]$	15	22.5	30	7.5
	x_1	-1	0	$+1$	
Wire cutting speed	v $[mm/min]$	400	500	600	100
	x_2	-1	0	$+1$	
Current intensity	I $[A]$	1,5	2	2,5	0.5

With the experimental planning, the effect of one factor often depends on the level of another factor, meaning there is an interaction effect between two factors. In such cases, the model becomes nonlinear.

$$\tilde{y} = b_0 + b_1 \cdot X_1 + b_2 \cdot X_2 + b_3 \cdot X_3 + b_{11} \cdot X_1^2 + b_{22} \cdot X_2^2 + b_{33} \cdot X_3^2 + b_{12} \cdot X_1 X_2 + b_{13} \cdot X_1 X_3 + b_{23} \cdot X_2 X_3, \quad \ldots (1)$$

where $y \rightarrow y_1$ *and* y_2 are measured parameters: external and internal measure,
$b_0, b_1, b_2, \ldots b_{23}$ are regression parameters, and
X_1, X_2, X_3 are model factors – density, wire cutting speed and current intensity, respectively.
Due to the interaction effect between two factors, the Box-Behnken model is used. The Box-Behnken model is a type of experimental design used to identify optimal conditions based on a selected number of factors, where each factor can have three levels. This design allows for efficient examination of the factors' influence on the experiment outcome, minimizing the number of required experimental points. Each factor in the experiment varies at three levels, enabling the study of factor effects under different conditions. Table 2 shows the experimental plan matrix, as well as the measurement results.

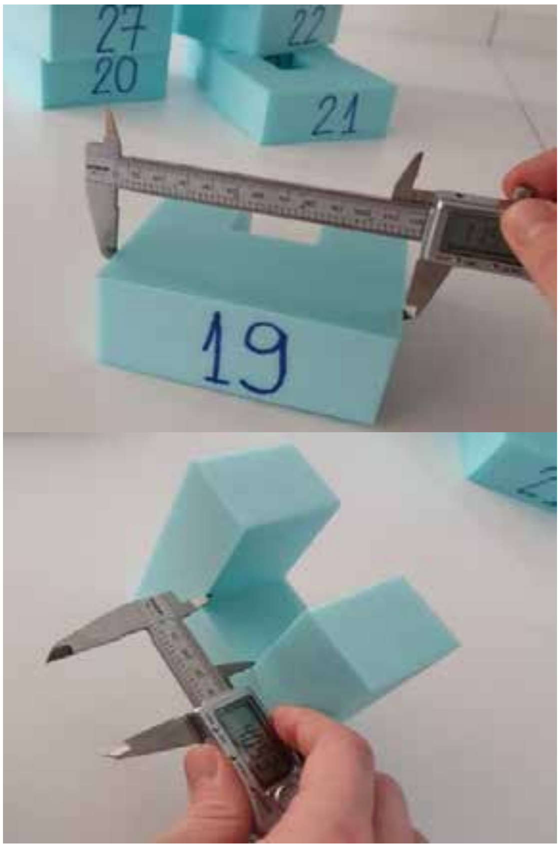

Figure 3 *Measurement of external and internal dimensions*

Table 2 The plan matrix of experiment and measured values

Exp. N^0	**Plan matrix**										**Measured values**	
	x_0	x_1	x_2	x_3	${x_1}^2$	${x_2}^2$	${x_3}^2$	x_1x_2	x_1x_3	x_2x_3	y_1	y_2
1	1	0	1	−1	0	1	1	0	0	−1	119.11	41.11
2	1	0	−1	−1	0	1	1	0	0	1	118.66	41.53
3	1	−1	0	−1	1	0	1	0	1	0	118.7	41.51
4	1	1	1	0	1	1	0	1	0	0	118.81	40.99
5	1	−1	−1	0	1	1	0	1	0	0	117.2	43
6	1	1	0	1	1	0	1	0	1	0	118.7	40.99
7	1	−1	0	1	1	0	1	0	−1	0	117.1	42.31
8	1	−1	1	0	1	1	0	−1	0	0	117.43	42.23
9	1	0	0	0	0	0	0	0	0	0	117.33	42.4
10	1	1	0	−1	1	0	1	0	−1	0	119.57	40.44
11	1	0	0	0	0	0	0	0	0	0	117.8	42.6
12	1	0	−1	1	0	1	1	0	0	-1	117.41	42.83
13	1	0	1	1	0	1	1	0	0	1	118.18	42.41
14	1	0	0	0	0	0	0	0	0	0	118	41.9
15	1	1	−1	0	1	1	0	−1	0	0	118.82	41.02

2.2. Experiment and measurement procedure

The management of the CNC styrofoam cutter with a hot wire is performed through special computer software that comes with the CNC cutter. This software enables the user to create or load the design they want to cut into the styrofoam, and to program the path along which the hot wire will cut the material. Once the programming is complete, the software sends commands to the cutter's control unit, which then executes the cutting according to the specified parameters. In this way, the CNC cutter utilizes the programmed path and motorized axes (x, y) for precise and repeatable cutting of materials such as styrofoam, allowing users to achieve high standards of quality and accuracy in their projects. The displayed CAD model of the desired styrofoam shape, previously drawn in AutoCAD software, is loaded into the CNC cutter software via a special file format, and the conditions and methods of the hot wire path, as well as the parameters of the styrofoam cutting process, are defined within the software.

After the completion of the cutting process of the designated types, shapes, and quantities of styrofoam pieces, the measuring procedure follows. Initially, the cut styrofoam pieces are marked and numbered. Once the cut styrofoam pieces are marked and numbered according to the type and density of styrofoam, the measurement of external and internal dimensions of the cut pieces follows. The measurement was conducted using a vernier caliper, as a measuring tool to determine the values of internal and external dimensions of the cut styrofoam pieces.

3. RESULTS AND DISCUSSION

When it comes to the experiment, two parameters were considered independently – the accuracy of external and internal measures, leading to a dual analysis of results and influential parameters. Three factors were taken into account: density, wire cutting speed, and current intensity, along with their squares and interactions. The hypothesis assumed the fundamental three parameters would exert the most significant influence on the experimental outcome. However, it was discovered that wire cutting speed was an entirely insignificant parameter, contradicting the initial hypothesis. A statistical analysis of the data was employed to evaluate the mathematical model. The data analysis was performed using the Data analysis tools package in Microsoft Excel and the results of the analysis are presented in Table 3 and Table 4, using Minitab 17.

The both tables display values such as the regression coefficient (R), coefficient of determination (R^2), standard errors, the sum of regression squares, and the F-test results for model adequacy. Given the dual analysis of results, the discussion will first address the analysis of outcomes related to the output parameter y_1, corresponding to the external measure.

The indicator of model adequacy is the regression coefficient, and if we look at this coefficient in both the internal and external measure, it exceeds the value of 0.95, which means that this process is very adequately described.

Table 3 Summary output for external measure - Regression Analysis

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.9759101			
R Square	0.9524005			
Adjusted R	0.8667214			
Standard Ei	0.2853857			
Observatio	15			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	9	8.148015	0.905335	11.11591
Residual	5	0.407225	0.081445	
Total	14	8.55524		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	117.71	0.164767513	714.4005	1.02E-13
ρ	0.68375	0.100899083	6.776573	0.001064
v	0.18	0.100899083	1.783961	0.134507
I	-0.58125	0.100899083	-5.76071	0.002213
ρ·ρ	0.26625	0.148519429	1.792695	0.133003
v·v	0.08875	0.148519429	0.597565	0.576162
I·I	0.54125	0.148519429	3.644304	0.014836
ρ·v	-0.06	0.142692852	-0.42048	0.691592
ρ·I	0.1825	0.142692852	1.278971	0.257048
v·I	0.08	0.142692852	0.560645	0.599231

Since three parameters are significant in regard to the external measure: density, current intensity and square of current intensity, we approach the optimization of the mathematical model that describes the influence of the given parameters, to achieve the goal of experiment.

The mathematical model that describes the influence of all parameters on the accuracy of the external measure is given by equation (...1). The analysis of measured data from Table 2 gave these models:

- for the external measure:

$$\tilde{y} = 118.188 + 0.68375 \cdot X_1 + 0.18 \cdot X_2 - 0.58125 \cdot X_3 + 0.26625 \cdot {X_1}^2 + 0.08875 \cdot {X_2}^2 + 0.54125 \cdot {X_3}^2 - 0.06 \cdot X_1X_2 + 0.1825 \cdot X_1X_3 + 0.18 \cdot X_2X_3 \quad \ldots(2)$$

- for the internal measure:

$$\tilde{y} = 41.818 - 0.70125 \cdot X_1 - 0.205 \cdot X_2 + 0.49375 \cdot X_3 - 0.57375 \cdot {X_1}^2 + 0.08375 \cdot {X_2}^2 - 0.41375 \cdot {X_3}^2 + 0.185 \cdot X_1X_2 - 0.0625 \cdot X_1X_3 + 0 \cdot X_2X_3. \quad \ldots(3)$$

Now that it has been determined which parameters are significant and which are not, in order not to overload the experiment and have more errors than benefits, it is necessary to exclude all insignificant parameters, so the given mathematical model is reduced to

$$\tilde{y} = 117.71 + 0.68375 \cdot X_1 - 0.58125 \cdot X_3 + 0.54125 \cdot {X_3}^2, \quad \ldots(4)$$

or after decoding the mathematical model

$$y_1 = 122.3 + 0.09 \cdot \rho - 5.5 \cdot I + 1.08 \cdot I^2 \quad \ldots(5)$$

Table 3 Summary output for internal measure - Regression Analysis

Regression Statistics				
Multiple R	0.9684693			
R Square	0.9379329			
Adjusted R	0.8262121			
Standard Ei	0.3291276			
Observatio	15			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	9	8.184815	0.909424	8.395328
Residual	5	0.541625	0.108325	
Total	14	8.72644		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	42.3	0.190021929	222.6059	3.47E-11
ρ	-0.70125	0.116364191	-6.02634	0.00181
v	-0.205	0.116364191	-1.76171	0.138418
I	0.49375	0.116364191	4.243144	0.008145
ρ·ρ	-0.57375	0.171283452	-3.34971	0.020335
v·v	0.08375	0.171283452	0.488956	0.645565
I·I	-0.41375	0.171283452	-2.41559	0.060446
ρ·v	0.185	0.164563817	1.124184	0.31199
ρ·I	-0.0625	0.164563817	-0.37979	0.719697
v·I	-1.78E-17	0.164563817	-1.1E-16	1

When it comes to the analysis of the results concerning the internal measure, unlike the previous case, the same parameters are not significant. In this case, they are: density, current intensity and square of density. So the given mathematical model is reduced to

$$\tilde{y} = 41.818 - 0.70125 \cdot X_1 + 0.49375 \cdot X_3 - 0.57375 \cdot {X_1}^2, \quad \text{... (6)}$$

or after decoding the mathematical model

$$y_2 = 36.7 + 0.36 \cdot \rho + 0.99 \cdot I - 0.01 \cdot \rho^2 \quad \text{... (7)}$$

Figures 4 and 5 show probability plots of the normal distribution of the residuals. We can observe the points are distributed approximately along the same direction and the connection is positive, i.e. the increase in the value of one variable follows the increase of another variable. The residuals obey a normal distribution.

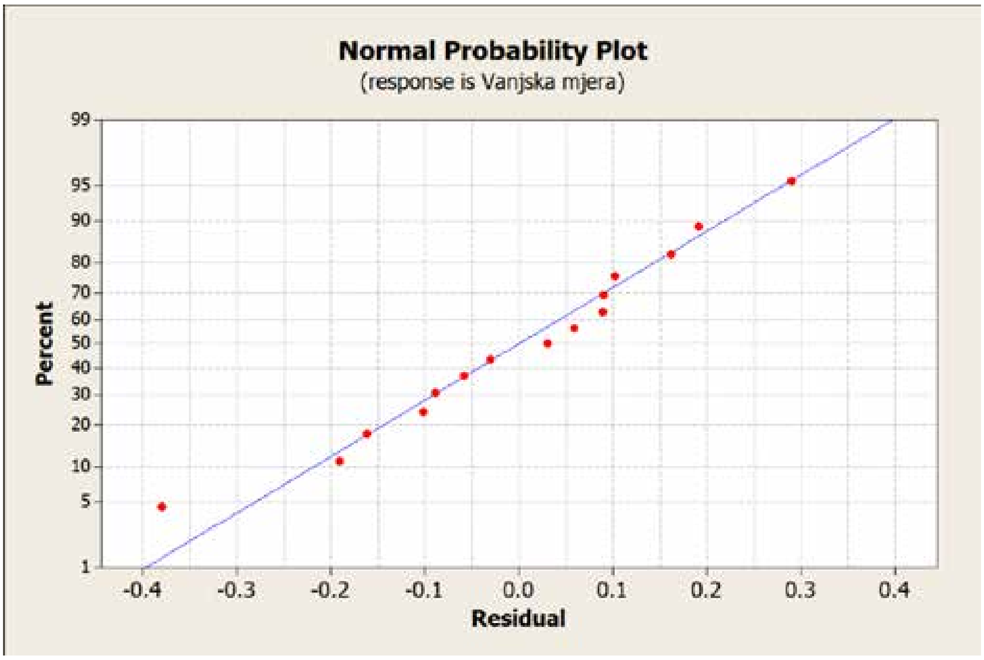

Figure 4 *Probability plot of normal distribution of residuals - Microsoft Excel (external measure)*

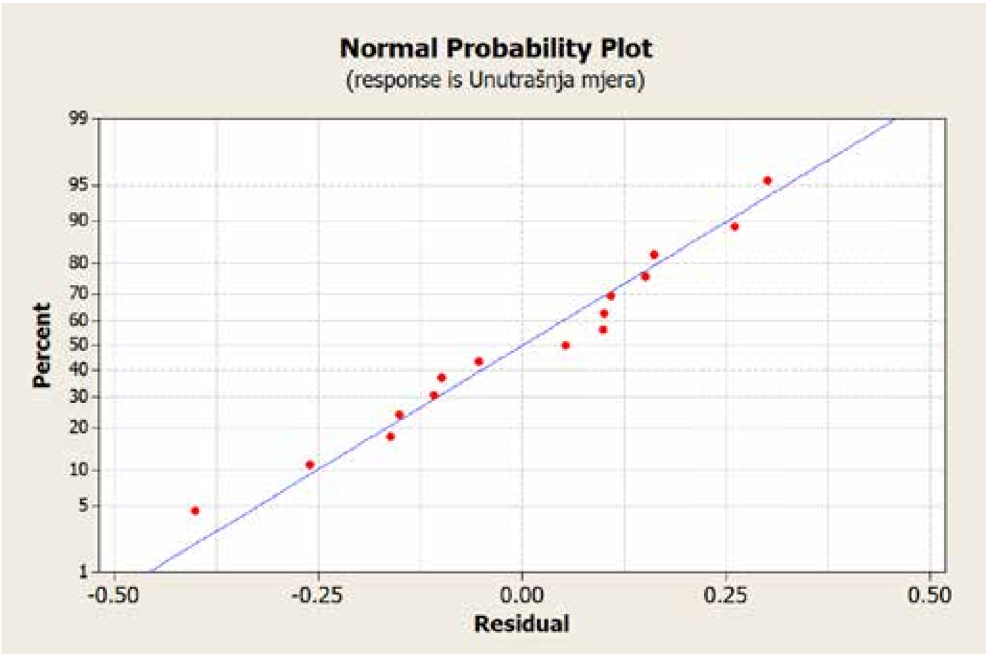

Figure 5 *Probability plot of normal distribution of residuals - Microsoft Excel (internal measure)*

4. CONCLUSION

The analysis of experiment's results, done by Microsoft Excel and Minitab 17, led to several concluding remarks, summarized in the following:

- By applying the planned experiment, the proccess of cutting styrofoam can be adequately described with the regression coefficient exceeding the value of 0.95 in both cases.
- The difference between these two output sizes is reflected in only one parameter – square parameter.
- Due to different significant parameters, we could not observe both output values at the same time, because in that case we would introduce a certain error, as we have proved.
- Optimizing the mathematical model significantly reduces the time of conducting the experiment, and as such it can be very easily applied to all future work and research.
- All other relevant quantities, such as the coefficient of determination and the correlation coefficient, were determined in accordance with the procedure, and as such show the validity of the conducted experiment.
- By introducing squares and interactions in the conducted experiment, we significantly increased the coefficient of determination, and in addition discovered an additional parameter that significantly contributes to the result.
- The final goal of the experiment was fulfilled, given that a mathematical model was obtained that describes the influence of the mentioned parameters.
- Taking all the above into account, the optimal combination of parameters for the accuracy of the external measurement is the maximum density, the minimum current strength and the mean value of the speed of the wire movement, and the same applies to the accuracy of the internal measurement.

In conclusion, the dual analysis of external measures sheds light on the varying influences of the considered parameters. The unexpected insignificance of wire cutting speed prompts a reevaluation of its role in the process. Further discussions will delve into the implications of these findings and potential adjustments to the experimental setup.

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