

# KOMBINOVANA EKSPERIMENTALNO-NUMERIČKA ANALIZA ELASTIČNIH LOPTI IZLOŽENIH NA PRITISAK IZMEĐU PLOČA

## COMBINED EXPERIMENTAL-NUMERICAL ANALYSIS OF ELASTIC BALLS SUBJECTED TO COMPRESSION BETWEEN PLATES

**Muamer Kargić**

Trendy d.o.o.  
Novi Travnik

**Aleksandar Karač**

Polytechnic Faculty  
University of Zenica

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**REZIME**

*U radu je predstavljen kombinovani postupak određivanja mehaničkih osobina elastičnih lopti ispunjenih fluidom pomoću testa na pritisak i odgovarajućeg analitičko-numeričkog rješenja. Korištene su obične dječje lopte, koje su modelirane Mooney-Rivlin modelom materijala. Dobiveni rezultati ovom kombinovanom numeričko-eksperimentalnom analizom upoređeni su s rezultatima testa na zatezanje. Dobivena su odlična slaganja u pogledu mjerenih i simuliranih veličina modula elastičnosti i debljine stijenke lopte.*

*Original Scientific paper*

**SUMMARY**

*This work presents the combined method for determination of mechanical characteristics of elastic balls filled with fluid by using compression test and appropriate analytical-numerical method. Ordinary playing balls, being modelled with the Mooney-Rivlin material model, are used in the study. Results obtained by this combined numerical-experimental analysis are compared with tensile test results. Very good agreements are achieved in terms of measured and simulated modulus of elasticity and ball thickness.*

### 1. UVOD

Pojam „elastična lopta“ u kontekstu ovog istraživanja predstavlja sferični tankostjeni objekat izrađen od (hiper)elastičnog materijala, koji je ispunjen fluidom. Da bi se odredile karakteristike materijala od kojih je izrađena takva lopta, uobičajeno je da se lopta razreže, odnosno uništi, iz nje naprave odgovarajuće epruvete, a zatim te epruvete ispituju koristeći neku od metoda ispitivanja (na primjer, test na zatezanje). Ovakva standardizirana ispitivanja s razaranjem često puta nisu moguća, jer broj takvih objekata može biti ograničen, objekti mogu biti skupi, ali i vrijeme ispitivanja može da igra ključnu ulogu. Također, objekat koji treba ispitati može biti potreban za dalje korištenje. [1]

### 1. INTRODUCTION

The term „elastic ball“ in this study is referred to a thin-walled spherical object made of (hyper)elastic material filled with fluid. In order to determine the material characteristics of such balls, it is common to cut the ball, ie. to destroy it, make specimens and test them using a test method (eg. tensile test). These standard destructive tests are not always possible due to limited number of objects, the objects may be expensive, or the time for testing may play a significant role. In addition, the object may be necessary for further use. [1]

Jedan od načina na koji se indirektno, bez razaranja, može doći do mehaničkih osobina materijala lopte (ali i dimenzionih) je kombinacija eksperimentalne i numeričke analize testa na pritisak. Na osnovu testa na pritisak moguće je dobiti zavisnost sile pritiska i pomjeranja pritisne ploče. S druge strane, pod pretpostavkom da se stijenka lopte može smatrati membranom, test na pritisak lopte između dvije ploče može se opisati pomoću dvije diferencijalne jednačine višeg reda, a ove jednačine se mogu riješiti numeričkim metodama [2-7]. Na taj način se upoređivanjem rezultata eksperimenta i numeričkog rješenja može doći do mehaničkih (i dimenzionih) osobina materijala, koji zadovoljavaju jednakost numeričkog i eksperimentalnog rješenja u smislu zavisnosti sila-pomjeranje.

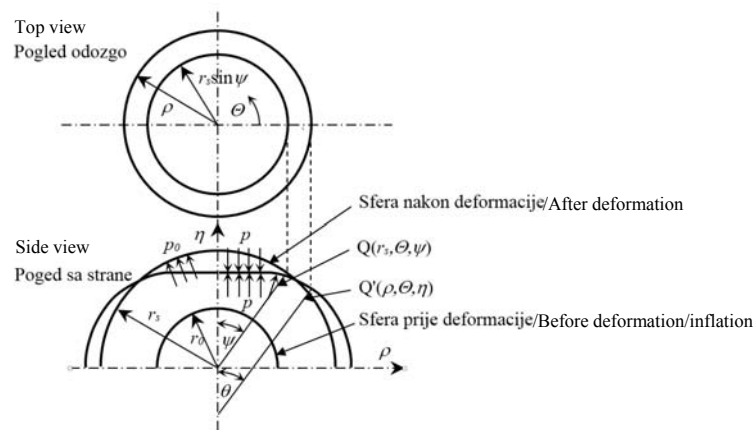
## 2. ANALITIČKO-NUMERIČKO RJEŠENJE

Analitički model, koji su razvili Feng i Yang [2, 3], bio je uvertira za detaljnija ispitivanja sfernih objekata opterećenih na pritisak [4-7], a problem je u ovom radu riješen pomoću MathCAD aplikacije [8]. Problem se svodi na analizu slučaja kontakta sferične membrane pod pritiskom i krute pritisne ploče, kao što je prikazano na slici 1.

One of the indirect non-destructive methods that can be used to obtain mechanical (and dimensional) characteristics of the ball material is combined experimental and numerical analysis of the compression test. Experimental compression test provides relationship between compression force and displacement of the compression plate. On the other hand, assuming the ball wall to be a membrane, the compression test can be described by a set of two higher-order differential equations that can be solved by numerical methods [2-7]. Thus, by comparing results between experiment and numerical solution, one can obtain a set of mechanical (and dimensional) characteristics of materials that satisfies the equality of both analyses in terms of force-displacement curve

## 2. ANALYTICAL-NUMERICAL SOLUTION

Analytical model, developed by Feng and Yang [2-3], was an overture to more detailed analyses of spherical objects under compression [4-7]. In this study, the problem is solved using a MathCAD application [8]. The problem description is given in Figure 1, where a pressurised spherical membrane is compressed with a rigid plate.



**Slika 1.** Geometrija problema pritiska sferične membrane između dvije ploče [2]  
**Figure 1.** Compression of the spherical membrane between plates – problem geometry [2]

Mogu se razlikovati dvije regije: kontaktna i beskontaktna, koje se mogu opisati sistemom od 5 kuplovanih diferencijalnih jednačina prvog reda s početnim uslovima [2,3]. Tako, za beskontaktnu regiju važi sljedeći sistem:

Two regions can be distinguished, contact and non-contact, that can be described by a set of five coupled first-order initial-value differential equations [2,3]. So, for the noncontact region there is a following set of equations:

$\lambda'_1 = \frac{\lambda_2 \cos \psi - \omega \left[ \frac{f_2(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)} \right] - \frac{\omega}{\lambda_2 \sin \psi} \left[ \frac{f_3(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)} \right]}{\omega = \delta' \text{ ili } \lambda'_2 = \frac{\omega - \lambda_2 \cos \psi}{\sin \psi}}$	(1)
$\omega = \delta' \text{ ili } \lambda'_2 = \frac{\omega - \lambda_2 \cos \psi}{\sin \psi}$	(2)
$\omega' = \frac{\lambda'_1 \omega}{\lambda_1} + \frac{\lambda_1 - \omega^2}{\delta} \left( \frac{T_2}{T_1} \right) - \frac{\lambda_1 \sqrt{\lambda_1^2 - \omega^2 p r_0}}{T_1}$	(3)

gdje su  $\lambda_1$  i  $\lambda_2$ , glavna izduženja u meridijalnom i obodnom pravcu, respektivno,  $\psi$  promjenljiva varijabla (ugao, v. Sliku 1.),  $\delta = \lambda_2 \sin \psi$ ,  $T_1$  i  $T_2$  glavni rezultantni naponi,  $p = P/C_1$ ,  $P$  pritisak unutar membrane (relativni u odnosu na spoljašnji pritisak),  $C_1$  karaktersitika materijala (v. dole),  $f_1, f_2, f_3$  su funkcije koje zavise od modela materijala membrane, odnosno glavnih rezultatnih napona:

where  $\lambda_1$  i  $\lambda_2$  are the principal stretches in meridian and circumferential directions, respectively,  $\psi$  is variable (angle, see Figure 1),  $\delta = \lambda_2 \sin \psi$ ,  $T_1$  i  $T_1$  principal stress resultants,  $p = P/C_1$ ,  $P$  is internal pressure (relative to the outer pressure),  $C_1$  is material parameter (see below),  $f_1, f_2, f_3$  are functions dependent on material model, ie. the principal stress resultants:

$f_1(\lambda_1, \lambda_2) = \frac{\partial T_1}{\partial \lambda_1}, \quad f_2(\lambda_1, \lambda_2) = \frac{\partial T_1}{\partial \lambda_2}, \quad f_3(\lambda_1, \lambda_2) = T_1 - T_2$	(4)
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Za kontaktnu regiju sistem jednačina (1)-(3) se reducira na dvije jednačine:

In the case of the contact region the system of Eqs.(1)-(3) reduces to the following two equations:

$\lambda'_1 = - \frac{\lambda_1 \left[ \frac{f_3(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)} \right] - \frac{\lambda_1 - \lambda_2 \cos \psi}{\sin \psi} \left[ \frac{f_2(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)} \right]}{\lambda'_2 = \frac{\lambda_1 - \lambda_2 \cos \psi}{\sin \psi}}$	(5)
$\lambda'_2 = \frac{\lambda_1 - \lambda_2 \cos \psi}{\sin \psi}$	(6)

Za kompletiranje sistema je neophodno odabrati model materijala i vrstu fluida, odnosno odgovarajući termodinamički zakon. U ovom radu korišten je dvo-parametarski Mooney-Rivlin model materijala, za koji su glavni rezultantni naponi:

To solve the system it is necessary to chose the material model and type of fluid, ie. thermodynamic law. Here, the two-parameter Mooney-Rivlin model is used for the ball material with the following principal stress resultants:

$T_1 = 2hC_1 \left( \frac{\lambda_1}{\lambda_2} - \frac{1}{\lambda_1^3 \lambda_2^2} \right) (1 + \alpha \lambda_2^2)$	(7)
$T_2 = 2hC_1 \left( \frac{\lambda_2}{\lambda_1} - \frac{1}{\lambda_1^3 \lambda_2^2} \right) (1 + \alpha \lambda_1^2)$	(8)

gdje je  $h$  debljina membrane,  $C_1 = E/[6(1+\alpha)]$ ,  $E$  modul elastičnosti,  $\alpha = C_2/C_1$ ,  $C_2$  drugi parametar materijala. Fluid (zrak) je stišljiv s izotermalnom promjenom,  $pV = \text{const.}$ , pri čemu je  $p$  apsolutni pritisak.

where  $h$  is the membrane thickness,  $C_1 = E/[6(1+\alpha)]$ ,  $E$  modulus of elasticity,  $\alpha = C_2/C_1$ ,  $C_2$  is the second material parameter. Fluid (air) is compressible with isothermal change,  $pV = \text{const.}$ , where  $p$  is the absolute pressure.

Postavljanjem odgovarajućih početnih uslova i uslova kontinuiteta, sistem jednačina (1)-(3) i (5)-(6) se rješava koristeći Runge-Kutta metodu četvrtog reda napisanu za ovu svrhu. Kao što je ranije rečeno, aplikacija je napisana u MathCAD softveru. Funkcije koje opisuju sistem diferencijalnih jednačina, a koje se koriste pri rješavanju Runge-Kutta metodom, date su na Slici 2 u MathCAD okruženju.

By setting appropriate initial and kinematic conditions, the system of equations is solved using fourth-order Runge-Kutta method programmed for this purpose.

As already said, the application is written in MathCAD software. Functions, describing the system of differential equations, that are solved by Runge-Kutta method, are given in Figure 2. in MathCAD interface.

The screenshot shows the MathCAD interface with a yellow header bar labeled "Differential equations". Below it, the text "Contact region equations" is followed by the definition of a function  $f(\psi, \lambda_1, \lambda_2)$  as a vector of two components. The first component is  $-\left(\frac{\lambda_1}{\lambda_2 \cdot \sin(\psi)}\right) \cdot f_3(\lambda_1, \lambda_2)$  and the second is  $-\left[\frac{\lambda_1 - \lambda_2 \cdot \cos(\psi)}{\sin(\psi)} \cdot \left(\frac{f_2(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)}\right)\right]$ . Below this, the function  $g(\psi, \lambda_1, \lambda_2)$  is defined as  $\frac{\lambda_1 - \lambda_2 \cdot \cos(\psi)}{\sin(\psi)}$ . The section "Non-contact region equations" follows, with  $f_2(\psi, \lambda_1, \lambda_2, w)$  defined as  $\frac{\lambda_2 \cdot \cos(\psi) - w}{\sin(\psi)} \cdot \frac{f_2(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)} - \frac{w}{\lambda_2 \cdot \sin(\psi)} \cdot \frac{f_3(\lambda_1, \lambda_2)}{f_1(\lambda_1, \lambda_2)}$ . The function  $g_2(\psi, \lambda_1, \lambda_2, w)$  is defined as  $\frac{w - \lambda_2 \cdot \cos(\psi)}{\sin(\psi)}$ . Finally,  $h_2(\psi, \lambda_1, \lambda_2, w, \text{dimlessP})$  is defined as  $\frac{f_2(\psi, \lambda_1, \lambda_2, w) \cdot w}{\lambda_1} + \frac{\lambda_1^2 - w^2}{\lambda_2 \cdot \sin(\psi)} \cdot \frac{T_2(\lambda_1, \lambda_2)}{T_1(\lambda_1, \lambda_2)} - \frac{\lambda_1 \cdot \sqrt{\lambda_1^2 - w^2} \cdot \text{dimlessP}}{2 \cdot \text{dimlessH} \cdot T_1(\lambda_1, \lambda_2)}$ . The bottom of the screenshot shows a small icon and the text "Differential equations".

**Slika 2.** Dio iz MathCAD aplikacije – definisanje diferencijalnih jednačina  
**Figure 2.** Section of the MathCAD application – definition of differential equations

Treba napomenuti da je u ovom istraživanju prethodno razvijena aplikacija za nestišljiv fluid [6] proširena modelom stišljivog fluida s izotermnom i adijabatskom promjenom. Također, implementiran je i algoritam koji uzima u obzir trenje između lopte i pritisnih ploča, kao u [7]. Proces je automatiziran, tako da je krajnji rezultat cjelokupna kriva sila-pomjeranje. Više detalja o samoj aplikaciji može se naći u [1].

### 3. EKSPERIMENTALNA ISPITIVANJA

Eksperimentalno istraživanje sastoji se od (i) mjerenja prečnika lopti, (ii) testa na pritisak i (iii) testova na zatezanje.

Prečnik lopti izmjeren je koristeći koordinatnu mjernu mašinu „ZEISS“ Contura G2 aktiv na

It should be mentioned that the previously developed application for incompressible fluid [6] is extended in this work to account for compressible fluid with isothermal and adiabatic change. An algorithm to account for friction between plates and ball is also implemented, as in [7]. The solution process is automated such that the final result is the full force-displacement curve of the test. More details about application can be found in [1].

### 3. EXPERIMENTAL STUDY

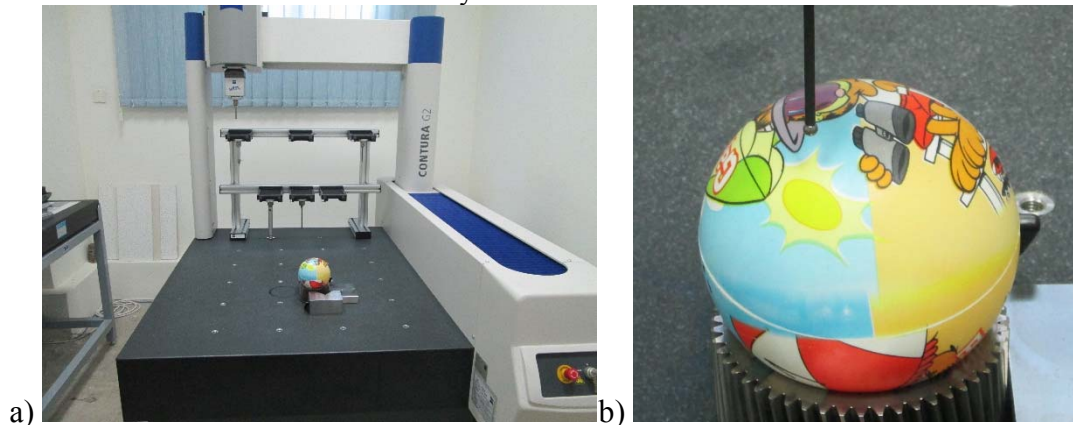
Experimental study consists of (i) measurements of the ball diameter, (ii) compression test and (iii) tensile tests.

Mašinskom fakultetu u Zenici (Slika 3.). Prije mjerenja prečnika lopti izmjeri se unutrašnji

pritisak pomoću uređaja za mjerenje pritiska tipa Fox. Na taj način dobija se vanjski prečnik na određenom pritisku, što predstavlja polazno stanje za test na pritisak, ali i ulazne podatke za numeričko rješavanje (prečnik i počeno izduženje).

The ball diameter is measured on coordinate machine ZEISS Contura G2 aktiv at Faculty of

Mechanical Engineering (Figure 3.). Prior to this measurement the pressure inside the ball is measured using a pressure device FOX. Thus, a diameter at a certain pressure is obtained, that is a starting point for the compression test, but also the input data for numerical solution (diameter and initial stretch).



**Slika 3.** Koordinatna mjerna mašina Zeiss Contura G2 aktiv i ispitivana lopta  
**Figure 3.** Coordinate measuring machine Zeiss Contura G2 aktiv and tested ball

Testovi na pritisak izvršeni su na mašini „ZWICK ROELL“ u firmi Austroterm d.o.o. u Bihaću. Nakon provjere pritiska i prečnika lopte, lopta se postavi između dvije pritisne ploče i vrši se pritisna deformacija brzinom 1 m/min. Pored praćenja sile i pomjeranja, na loptu je priključen i uređaj za mjerenje pritiska tipa Fox. Sekvenca testa prikazana je na Slici 4.

Compression tests are conducted on „ZWICK ROELL“ tensile machine at Austroterm company in Bihać. Prior to the test, diameter and pressure in the ball are checked and the ball is compressed at speed 1 m/min. Besides monitoring the force and compression displacement, the ball is equipped with the pressure device FOX to record the ball internal pressure during the test. A test sequence is given in Figure 4.



**Slika 4.** Test na pritisak s mjerenjem pritiska u lopti [1]  
**Figure 4.** Compression test with pressure measurement [1]

Kao rezultat mjerenja dobiva se kriva napon-deformacija, koju je neophodno konvertovati u krivu sila-pomjeranje, kao što je objašnjeno u [1].

As a result, one can obtain stress-strain relationship that needs to be converted into force-displacement curve, as explained in [1].

Također, pomoću video zapisa i očitavanja vrijednosti pritiska s uređaja za mjerenje pritiska u zavisnosti od vremena i/ili pomjeranja, dobiva se kriva pritisak-pomjeranje.

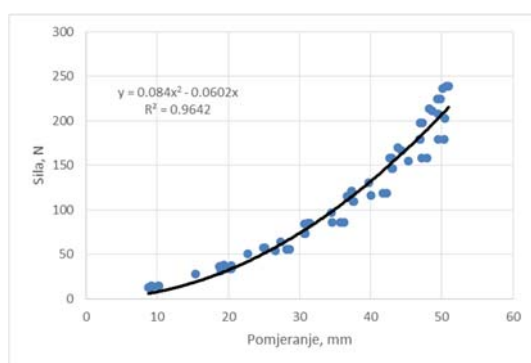
Prva kriva koristi se za fino podešavanje ulaznih podataka u numeričkoj simulaciji, dok se drugi dijagram koristi kao kontrola procesa određivanja karakteristika materijala lopti.

Na slici 5 dat je tipični dijagram sila-pomjeranje za jednu od ispitivanih lopti. Tačkasti podaci predstavljaju očitavanja iz nekoliko ponovljenih testova s odgovarajućom parboličkom aproksimacijom.

In addition, using the video recordings and pressure readings from THE pressure device (in time and/or compression displacement), one can obtain the pressure-displacement diagram.

The former curve is used for fine tuning of input parameters in numerical simulation, whereas the latter one is used to control the process for determination of ball characteristics.

Figure 5. shows a typical force-displacement diagram for a tested ball. Dots are individual pressure readings for several independent tests, shown together with parabolic regression curve.



**Slika 5.** Zavisnost sila-pomjeranje za test na pritisak  
**Figure 5.** Force-displacement relationship for compression test

Ispitivanje na zatezanje predstavlja verifikaciju kombinovane eksperimentalno-numeričke procedure i biće detaljnije obrađeno u dijelu 5.

Tensile test is used for verification of the combined experimental-numerical procedure and will be explained in more detail in Section 5.

#### 4. UTVRĐIVANJE KARAKTERISTIKA MATERIJALA

Ulazni podaci za numeričku analizu pomoću MathCAD aplikacije su: prečnik lopte, modul elastičnosti, debljina stijenke lopte, početno izduženje, broj proračunskih tačaka i kontaktni ugao. Početno izduženje predstavlja odnos između prečnika u napuhanom i nenapuhanom stanju, dok se kontaktni ugao definiše kao ugao čiji luk čini dio membrane u kontaktu s pritisnom pločom posmatran u nedeformiranom stanju. Vrijednosti početnih podataka za jednu od lopti dati su u Tabeli 1.

#### 4. DETERMINATION OF MATERIAL CHARACTERISTICS

Input data for the MathCAD application are: ball diameter, modulus of elasticity, wall thickness, initial stretch, number of computational points and contact angle. The initial stretch is the ratio between the diameter of the inflated ball and that of the non-inflated one, whereas the contact angle is the angle of the membrane arc that is in contact with the compression plate in inflated state prior to the compression. Parameter values for a tested ball are given in Table 1.

**Tabela 1.** Ulazni podaci za numeričku simulaciju  
**Table 1.** Input parameters for numerical simulation

Veličina/Parameter	Vrijednost/Value	Veličina/Parameter	Vrijednost/Value
Prečnik lopte, mm Ball diameter, mm	93.72	Broj proračunskih tačaka Number of computational points	300
Modul elastičnosti, MPa* Modulus of elasticity, MPa*	4 (3.2)	Kontaktni ugao, °* Contact angle, °*	44 (53.5)
Debljina stijenke, mm* Ball thickness, mm*	1.5 (1.45)	Početno izduženje, - Initial extension, -	1.05

\*veličine koje se mijenjaju/Parameters to be modified

Pored pomenutih podataka, postavlja se i model materijala. U ovom slučaju korišten je Mooney-Rivlin model, ali postoji mogućnost korištenja tro-parametarskog Mooney-Rivlin modela, opšteg Mooney modela [9,10] te Hooke-ovog modela. Drugi parametar korištenog modela je zanemaren ( $\alpha=0$ ), tako da je u osnovi korišten neo-Hookeov model. Također, postavlja se termodinamički zakon za fluid u lopti – koristi se izotermalni,  $pV=\text{const}$ , ali postoji mogućnost odabira i adijabatske promjene  $pV^\kappa=\text{const}$ , za stišljive fluide, te nestišljiv fluid,  $V=\text{const}$ .

U radu [1] je izvršena analiza uticaja pojedinih promjenljivih parametara na rezultate analize. Ustanovljeno je da na silu utiču svi parametri, ali je od pomentih parametara uticaj ugla  $\alpha$  najveći, dok ostali parametri imaju ulogu 'dotjerivanja'. Na stepen kompresije najviše utiče ugao  $\alpha$ , donekle početno izduženje, a ostali parametri imaju mali ili nikakav uticaj. Na odnos krajnjeg i početnog pritiska najznačajniji uticaj ima ugao  $\alpha$ , dok je uticaj ostalih parametara veoma mali ili nikakav.

Također je pokazano da koeficijent trenja od vrijednosti 0 (klizanje bez trenja) do kritične vrijednosti (nema klizanja) ima neznatan uticaj na rezultate analize [1], pa se u ovom radu neće analizirati.

Mijenjajući podatke za modul elastičnosti, debljinu stijenke i kontakti ugao, dolazi se do različitih krivih sila-pomjeranje, s krajnjim ciljem da se dobije dobro slaganje s eksperimentalnim rezultatima. Na Slici 6.a) prikazane su eksperimentalna i odgovarajuća numerička kriva nakon podešavanja parametara. Vrijednosti koje zadovoljavaju prethodni proračun su dati u Tabeli 1. u zagradama. Kao kontrola procesa upoređen je i unutrašnji pritisak. Zavisnost promjene pritiska od pomjeranja data je na Slici 6b.

Besides aforementioned parameters, a material model is also set. Mooney-Rivlin model is used here, but there is option to use three-parameter Mooney-Rivlin, general Mooney [9,10] and Hookean model. The second model parameter is neglected ( $\alpha=0$ ), so, basically, the model used is neo-Hookean. The thermodynamic law for the fluid inside the ball is set to be isothermal,  $pV=\text{const}$ , but one can also use adiabatic,  $pV^\kappa=\text{const}$ , for compressible fluids, and incompressible model,  $V=\text{const}$ .

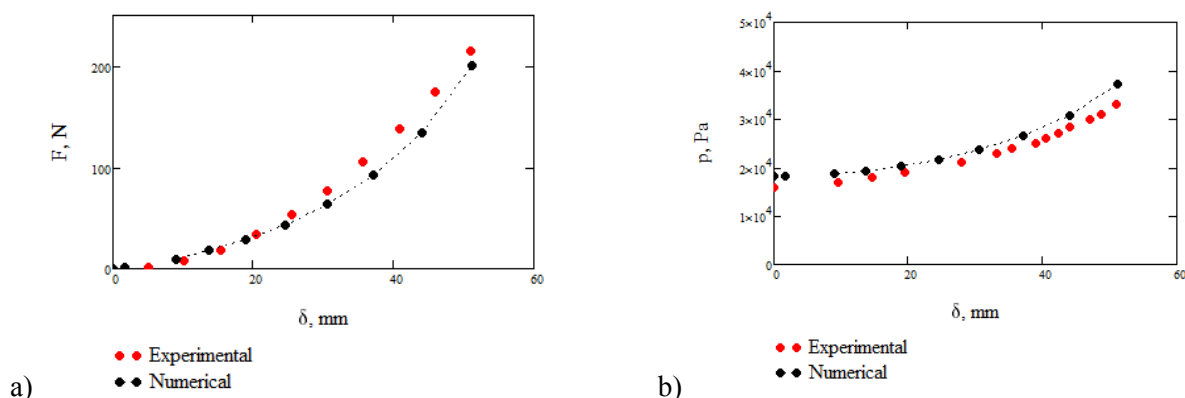
The detailed analysis of the influence of aforementioned parameters on results is given in [1]. It is shown that all parameters affect the force, but the most prominent is the contact angle. Other parameters are usually used for fine-tuning. Compression ratio is most affected by the contact angle and somewhat by initial stretch, whereas the influence of other parameters can be neglected. The ratio between final and initial pressure is most affected by the contact angle with other parameters having small influence if any.

It is also shown that the friction between plates and the ball, in the range of friction coefficient between 0, ie. slip condition, and the critical value, ie. no-slip condition, has negligible influence on the results [1] and will not be analysed here.

By varying the values of modulus of elasticity, wall thickness and contact angle, one can obtain different load-displacement curves, with the aim to obtain a good agreement with the experimental results. Figure 6a shows the experimental and corresponding numerical curve after fine-tuning of the parameters. Parameter values that satisfy the final calculations are given in Table 1 (see values in brackets). As a procedure control, the internal ball pressure is also compared with the comparison, as given in Figure 6b.

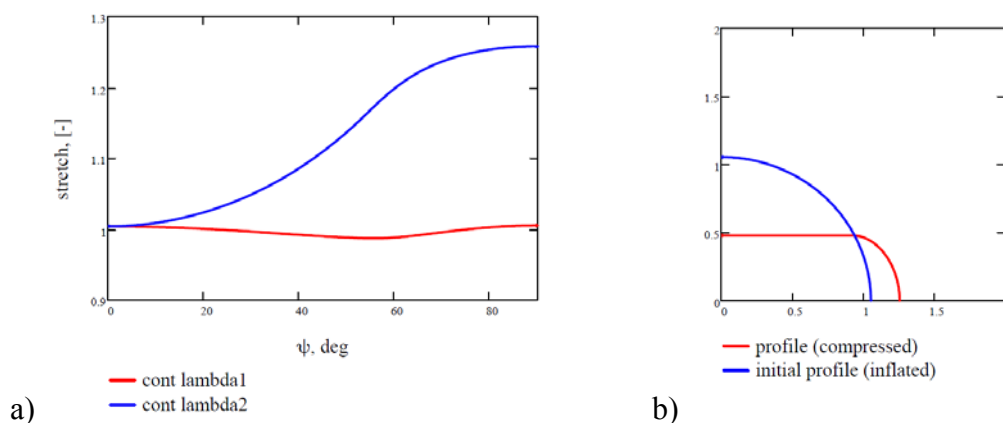
Treba napomenuti da se u MathCAD aplikaciji dobivaju i vrijednosti izduženja/deformacija i napona u dva glavna pravca, te profil lopte u toku testa (Slika 7).

It has to be mentioned that the MatchCAD application also provides the stretch/deformation and stress histories in the two principal directions and ball profile (Figure 7).



**Slika 6.** Uporedni rezultati eksperimenta i numeričke analize: a) sila-pomjeranje, b) pritisak-pomjeranje

**Figure 6.** Comparison between experimental and numerical results: a) force-displacement, b) pressure-displacement



**Slika 7.** Dodatni rezultati numeričke analize: a) glavna izduženja, b) profil lopte

**Figure 7.** Additional numerical results: a) principal stretches, b) ball profile

## 5. VERIFIKACIJA REZULTATA

Verifikacija rezultata eksperimentalno-numeričke analize urađena je pomoću testova na zatezanje. Iz materijala lopti su izrađene epruvete prema standardu ASTM D 638-03 [9] i ispitivane na mašini „Zwick Roell“ u firmi Car Trim d.o.o. u Žepču. Prije ispitivanja izmjerene su debljine stijenki za sve epruvete.

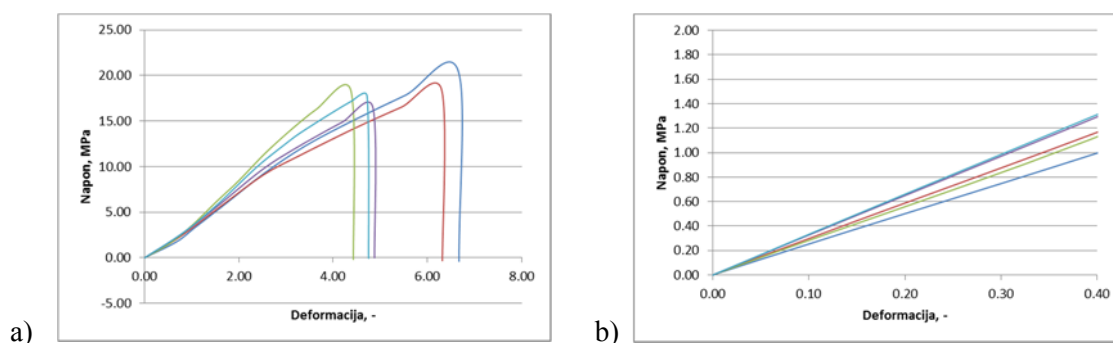
Na slici 8. date su inženjerske krive napon-deformacija za ispitivanu loptu. Imajući u vidu rezultate numeričke analize, gdje se pokazalo da izduženja ne prelaze vrijednosti 1.4, u procjeni modula elastičnosti uzimaju se samo deformacije do 40% (Slika 7b).

## 5. VERIFICATION OF RESULTS

Verification of the results of this combined experimental-numerical analysis is carried out via tensile tests. Specimens are cut from the ball material according to the ASTM D 638-03 [9] and tested on the „ZWICK ROELL“ machine at the company Car trim d.o.o. company in Žepče. Prior to testing the thickness for all specimens is measured.

Figure 8. gives engineering curves for the ball tested. Having in mind numerical results, with stretches below 1.4, only the first part of the curves is considered with deformations below 40% (Fig. 7b).





**Slika 8.** Inženjerske krive napon-deformacija za materijal ispitivane lopte  
**Figure 8.** Engineering stress-strain curves for the ball material

U svrhu dobivanja relevantnijih podataka, debljina stijenke lopte mjerena je i na ostalim dijelovima isječenih lopti. Za posmatranu loptu dobivene su vrijednosti debljine stijenke od 1.52 mm i modula elastičnosti od 3.1 MPa.

## 6. ZAKLJUČCI

Uzimajući u obzir kvalitativne rezultate u pogledu krivih sila-pomjeranje i pritisak-pomjeranje (Slika 6), te kvantitativnih vrijednosti za modul elastičnosti (3.2 MPa and 3.1 MPa) i debljinu stijenke (1.45 mm i 1.52 mm), može se zaključiti da je metod pouzdan za procjenu mehaničkih osobina sfernih elastičnih objekata. Također, pokazano je da Mooney-Rivlin/neo-Hookean model materijala uspješno opisuje ponašanje materijala od kojih su lopte napravljene.

Treba, ipak, napomenuti da je uvijek neophodno izvršiti dodatnu kontrolu eksperimenta, s obzirom da različite kombinacije ulaznih parametara u numeričkoj simulaciji mogu dovesti do istog rješenja u pogledu krive sila-pomjeranje. U ovom radu to je urađeno praćenjem pritiska unutar lopte za razliku od uobičajenog pristupa u literaturi, tj. video praćenja kontaktne površine između lopte i pritisne ploče. Ukoliko dodatni test nije moguće zadovoljiti, postoji mogućnost da model materijala nije prikladan. Ovo je pokazano u radu [1] gdje Mooney-Rivlin model nije doveo do dobrog slaganja krive pritisak-pomjeranje, iako su se eksperimentalna i numerička kriva sila-pomjeranje odlično slagale. Umjesto Mooney-Rivlin modela implementirao se i primijenio opšti Mooney model materijala [10,11] sa zadovoljavajućim rezultatima. Ovo istraživanje je trenutno u toku.

To obtain more convenient results the wall thickness is measured at other parts of the ball material. For the ball analysed in this paper measured wall thickness is 1.52 mm, and modulus of elasticity evaluated from the Figure 5b is 3.1 MPa.

## 6. CONCLUSIONS

Taking into consideration qualitative results in terms of the load-displacement and pressure-displacement curves (Figure 6.) and quantitative results for the modulus of elasticity (3.2 MPa vs. 3.1 MPa) and wall thickness (1.45 mm vs. 1.52 mm), one can conclude that the presented method is reliable tool for determination of mechanical characteristics of the spherical elastic objects. It is also demonstrated that the Mooney-Rivlin/neo-Hookean material model successfully described the ball material behaviour. However, it has to be noted that the additional control tests are necessary, since different combinations of input data can lead to the same results in terms of load-displacements curves. In this work the control is carried out via monitoring internal pressure unlike the common procedure in literature, where the contact area is recorded using video equipment. If the additional control test is not possible, there is a possibility that the material model applied is not correct. This is demonstrated in [1], where the application of Mooney-Rivlin material model does not lead to the satisfactory pressure-displacement curve, although experimental and numerical force-displacement curves are in agreement. Instead, the general Mooney material model [10,11] is implemented and applied with satisfactory results. This research is currently in progress.

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### Corresponding author:

**Aleksandar Karač**

**Polytechnic Faculty, University of Zenica**

**Email: akarac@ptf.unze.ba**

**Phone: +387 32 44 44 20**