MATEMATIČKA I NUMERIČKA ANALIZA ODNOSA IZMEĐU LOSCHMIDTOVE KONSTANTE, AVOGADROVE KONSTANTE I BRZINE ZVUKA U REALNIM PLINOVIMA KOD RAZLIČITIH TERMODINAMIČKIH SVOJSTAVA (pvT)

MATHEMATICAL AND NUMERICAL ANALYSIS OF THE RELATIONSHIP AMONG THE LOSCHMIDT CONSTANT, THE AVOGADRO CONSTANT, AND THE SPEED OF SOUND IN REAL GASES AT DIFFERENT THE *pvT* THERMODYNAMIC PROPERTIES

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REZIME:

U ovom radu izvedena je diferencijalna jednadžba koja daje odnos između Loschmidtove konstante, Avogadrove konstante i brzine zvuka u stvarnim plinovima kao funkcije termodinamičkih svojstava plina (tlak, temperatura i volumen). Konstantni omjeri NL/NA i relativno odstupanje omjera NL/NA od srednje vrijednosti za plinove (vodik, helij, ugljenmonoksid, argon, dušik, kisik, ugljendioksid i metan) izračunati su korištenjem njihovih stvarnih jednadžbi stanja, potvrđujući valjanost izvedene jednadžbe. Prvi izračun omjera konstanti NL/NA i relativnog odstupanja omjera NL/NA od srednje vrijednosti za plinove u standardnom stanju (temperatura 273,15 K i tlak 101,325 Pa) pokazuje dobro slaganje s objavljenim rezultatima u literaturi, s obzirom na to da je literaturni podatak omjera NL/NA 0,04461498 u standardnom stanju. Drugi izračun konstantnih omjera NL/NA i relativnog odstupanja omjera NL/NA od srednje (prosječne) vrijednosti za stvarne plinove napravljen je za tlak od 500 000 Pa i temperaturu od 293,15 K, što je također rezultiralo sličnim ponašanjem kao u prethodnom slučaju. Treći izračun konstantnih omjera NL/NA i relativnog odstupanja omjera NL/NA od srednje (prosječne) vrijednosti za stvarne plinove napravljen je za tlak od 1000 000 Pa i temperaturu od 320,0 K, što također daje slično ponašanje kao u prethodnom slučaju.

Short original scientific paper

SUMMARY:

In this paper, a differential equation is derived that gives the relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound in real gases as a function of the thermodynamic properties of the gas (pressure, temperature, and volume). The constant ratios N_L/N_A *and the relative deviation of the* N_L/N_A *ratio from the mean value for the gases (hydrogen, helium, carbon monoxide, argon, nitrogen, oxygen, carbon dioxide, and methane) were calculated using their real-equations of state, confirming the validity of the derived equation. The first calculation of the ratio of the constants* N_L/N_A *and the relative deviation of the* N_L/N_A *ratio from the mean value for the gases at standard state (temperature 273.15 K and pressure 101,325 Pa) shows a good agreement with the published results in the literature, given the fact that the literature data of the* N_L/N_A *ratio is 0.04461498 at standard state. The second calculation of the constant ratios* N_L/N_A *and the relative deviation of the* N_L/N_A *ratio from the mean (average) value for the real gases was made for a pressure of 500,000 Pa and a temperature of 293.15 K, which also resulted in similar behavior as in previous case. The third calculation of the constant ratios* N_L/N_A *and the relative deviation of the* N_L/N_A *ratio from the mean (average) value for the real gases was made for a pressure of 1000,000 Pa and a temperature of 320.0 K, which also gave similar behavior as in previous case.*

1. INTRODUCTION

In 1811, an Italian professor of physics *Amadeo Carlo Avogadro* suggested an important hypothesis 'that equal volumes of all gases at the same temperature and pressure contain the same number of molecules', or 'the volume of a gas at a given pressure and temperature is proportional to the number of atoms or molecules regardless of the nature of the gas', which is well known as the Avogadro's Principle (law) or Avogadro's constant (number). The greatest problem Avogadro had to resolve was the confusion at that time regarding atoms and molecules. One of his most important contributions was clearly distinguishing one from the other, stating that gases are composed of molecules, and these molecules are composed of atoms. Avogadro did not actually use the word 'atom' as the words 'atom' and 'molecule' were used almost without difference. He believed that there were three kinds of 'molecules', including an 'elementary molecule' ('atom') [1]. Thus, the hypothesis has been extremely visionary, and her confirmation, using the kinetic theory of gases, came decades later.

The scientific community did not give great attention to his theory, so Avogadro's hypothesis was not immediately accepted. The studies by *Charles Frédéric Gerhardt* and *Auguste Laurent* on organic chemistry made it possible to demonstrate what Avogadro's law explained, i.e. why the same quantities of molecules in a gas have the same volume. Unfortunately, related experiments in organic substances showed seeming exceptions to the law. This was finally resolved by *Stanislao Cannizzaro,* as announced at the Karlsruhe Congress in 1860, four years after Avogadro's death. He explained that these exceptions were due to molecular dissociations at certain temperatures, and that Avogadro's law determined not only molecular masses, but atomic masses as well. In 1911, a meeting in Turin commemorated the hundredth anniversary of the publication of Avogadro's classic 1811 paper. Thus, Avogadro's great contribution to chemistry was recognised. *Rudolf Clausius*, with his kinetic theory on gases, gave another confirmation of Avogadro's Law. *Jacobus Henricus van 't Hoff* showed that Avogadro's theory also applied to dilute solutions. Avogadro is hailed as a founder of the atomic-molecular theory [1].

Despite the fact that Avogadro did not specify the ratio of the number of constituent particles in a sample to the amount of substance, the French physicist *Jean Baptiste Perrin* (1909) proposed naming the constant in honor of Avogadro. *Jean B. Perrin* won the Nobel Prize in Physics (1926) in a large part for his work in determinig the Avogadro constant. Perrin's method was based on the Brouwnian motion [2].

In the years since then, several different methods (coulometry, electron mass measurement, x-ray crystal density method) have been used to estimate the magnitude of this fundamental constant. In general, accurate determinations of Avogadro's number require the measurement of a single quantity on both the atomic and macroscopic scales using the same unit of measurements. This became possible for the first time when physicist *Robert Milllikan* measured the charge on an electron in 1910. The charge of a mole of electrons had been known since 1834 when *Michael Faraday* published his works on electrolysis.The charge was called the Faraday constant, and the best value of the constant is 96,485.3383 (3) C/mol, according to the NIST. The best estimate of the charge on an electron, based on modern experiments, is $1.60217653\cdot10^{-19}$ C/electron. When divide the charge on a mole of electrons by the charge on a single electron we obtain a value of Avogadro's number of $6.02214154\cdot10^{23}$ particles/mole [3]. Since 1910, newer calculations have more accurately determined the value for Farady's constant and elementary charge. Another approach to determining Avogadro's number starts with careful measurements of the density of an ultrapure sample of a material on the macroscopic scale.

In fact, Perrin originally proposed the name Avogadro's number to refer to the number of molecules in one gram-molecule of oxigen, and this term is still widsely used in introductory works. The change of name to the Avogadro constant came with the introduction of the mole as a unit in the International System of Units (SI) in 1971, which recognised amount of substance as an independent dimension of measurements. Thus, the Avogadro constant was no longer a pure number, now it had a unit of measurements, the reciprocal mole (mol^{-1}) [4, 5].

The Avogadro constant is a scaling factor between macroscopic and microscopic observations of nature, and can be applied to any substance. The most significant consequence of Avogadro's law is that the gas constant has the same value for all gases. Because of its role as a scaling factor, it provides the relation between other physical constants and properties. For example, it establishes a relationship between: ■ the universal gas constant (R_u) and the Boltzmann constant (k_B): $R_u=k_B \cdot N_A$=8.314472 (15) J/mol K; ■ the Faradey constant (F) and the elementary charge (e): $F=N_A \cdot e$=96,485.3383 (3) C/mol, and ■ the Avogadro constant and the definition of the unified atomic mass unit (u): 1 $u=M/N_A$= $1.660538782(83) \cdot 10^{-24}$ g.

The Loschmidt number is defined as the number of atoms in a gram-atom or the number of molecules in a gram-molecule. This number, in literature, is frequently referred to as the Avogadro's number. But, the term Loschmidt number is reserved for the number of molecules in a cubic centimeter of a gas under standard conditions. In German language literature may refer to both constants by the same name, distinguished only by the units of measurement. The first actual estimate of the number of molecules in one cubic centimeter of a gas under standard conditions was made in 1865 by the Austrian physicist *Johan Josef Loschmidt*, professor at the University of Vienna. The number density of particles in a gas is now called the Loschmidt constant in his honor, and is approximately proportional to the Avogadro constant. Loschmidt's method was based on the kinetic theory of gases. The kinetic theory had been developed with great success by the efforts of two scientists *James Clerk Maxwell* and *Rudolph Clausius*. Loschmidt was the first to estimate the physical size of molecules in 1865, but he did not actually calculate a value for the constant which now bears his name. Loschmidt number is, by virtue of its definition, the same for atoms and molecules of all kinds. Though molecules may vary in size, shape and mass, the number of molecules in a gram-molecule is a universal constant for all solids, liquids and gases, elements and compounds [6].Thus, the number of molecules in a specific volume of gas is independent of the size or mass of the gas molecules. As an example, equal volumes of molecular hydrogen and nitrogen, as ideal gasous, would contain the same number of molecules, as long as they are at the same temperature and pressure.

The Loschmidt constant is usually quoted at standard state of substance (i.e., temperature T_0=273,15K and pressure p_0=1atm=1.01325bar=101325 Pa), and recommended value is $2.6867774(47) \cdot 10^{25}$ particles/m^3. The pressure and temperature can be chosen freely, and must be quoted with values of the Loschmidt constant. The precision to which the Loschmidt constant is currently known is limited entirely by the uncertainty in the value of the gas constant.

The sound waves are a result of the movement of the elastic pieces of the substance environment whether it is a substance in the gas, liquid or solid. A change in gas density (or pressure) is transmitted in all directions with a certain speed. Number of compression and expansion of the environment, performed by the sound source by a sinuous curve in the unit of time is called ***frequency*** – unit is the number of cycles per second (c/s). The distance between the two maximum and minimum in a sinuous curve represents ***wavelength*** – unit is meters (m). Pressure changes represent ***sound pressure*** – the unit is N/m^2. Sound energy (J) per unit time

(s) passing through a unit area (m^2) normal to the direction of propagation is called ***the intensity of sound waves*** or alternatively ***the power density*** – the unit is W/m^2 [7]. The quantity has practical importance. The human ear is a very sensitive organ and at the same time it is very flexible. The lower limit of the audible intensity of sound is of the order of 10^{-12} W/m^2 and the maximum safety limit is the order of 1 W/m^2. The sound waves intensity of ordinary conversation is the order of 10^{-6} W/m^2, street traffic is 10^{-5} W/m^2 and jet plane is 10^{-2} W/m^2.

Propagation speed of sound waves depends only on the properties of the medium through which the propagation takes place. The speed of sound varies from substance to substance, for instance: sound travels most slowly in gases (the average gas speed of sound is about 330 m/s); it travels faster in liquids (the average gas speed of sound is about 1,500 m/s); and it travels fastest in solids (the average gas speed of sound is about 4,000 m/s). In an exceptionally stiff material, such as diamond, sound travels at 12,000 m/s - which is around the maximum speed that sound will travel under normal conditions. In common everyday speech, speed of sound refers to the speed of sound waves in air. At 20°C the speed of sound in air is about 343 m/s. The speed of sound in an ideal gas depends only on its temperature and composition. The speed of sound has a weak dependence on frequency and pressure in ordinary air, deviating slightly from ideal behavior.

2. MATHEMATICAL ANALYSIS

2.1. The thermodynamic speed of sound

The thermodynamic speed of sound (i.e., the speed of sound at zero frequency) in a fluid u, *m/s* is defined by the Laplace equation

$$u^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \tag{1a}$$

where: ρ, kg/m^3 is the density of the substance; p, N/m^2 is the pressure; s, $J/kg{\cdot}K$ is the specific entropy of the substance. Since $\rho = 1/v$, the Laplace equation has the following form, according to Ref. [8, p.127]:

$$u^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_s \tag{1b}$$

By combining the above equation with the important relationship that determines the following derivative, rarely mentioned in the literature, according to Ref. [8, p.124]:

$$\left(\frac{\partial p}{\partial v}\right)_s = \left(\frac{\partial p}{\partial v}\right)_T - \frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 \tag{2}$$

it is obtained that

$$u^2 = v^2\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 - \left(\frac{\partial p}{\partial v}\right)_T\right] \tag{3}$$

An equivalent form of Eq. (3) can be found by replacing the derivative $(\partial p/\partial T)_v$ in terms of the cyclic equation, according to Ref. [9, p.636]:

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T \tag{4}$$

so that Eq. (3) results

$$u^2 = v^2 \left[\frac{T}{c_v} \left(\frac{\partial p}{\partial v} \right)_T^2 \left(\frac{\partial v}{\partial T} \right)_p^2 - \left(\frac{\partial p}{\partial v} \right)_T \right] \tag{5}$$

or

$$v = u \left[\frac{T}{c_v} \left(\frac{\partial p}{\partial v} \right)_T^2 \left(\frac{\partial v}{\partial T} \right)_p^2 - \left(\frac{\partial p}{\partial v} \right)_T \right]^{-\frac{1}{2}} \tag{5a}$$

2.2. Relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound

The Loschmidt constant is related to the Avogadro constant by relation, according to Ref. [10, p. 418]:

$$N_L = \frac{p}{R_u T} N_A = \frac{\rho(p,T)}{M} N_A = \frac{1}{v(p,T) \cdot M} N_A, \quad \frac{\text{particles}}{\text{m}^3}, \tag{6}$$

where: N_L, particles/m^3 is the Loschmidt constant; p, N/m^2 is the pressure; T, K is the temperature; R_u, J/kmol·K is the universal gas constant; ρ (p, T), kg/m^3 is the density of the substance; v (p, T), m^3/kg is the specific volume of the substance; M, kg/kmol is the atomic mass of the substance, and N_A, particles /kmol is the Avogadro constant.

Combining Eq. (5a) and (6), the relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound, is obtained in the following form:

$$\frac{N_L}{N_A} = \frac{1}{u \cdot M} \left[\frac{T}{c_v} \left(\frac{\partial p}{\partial v} \right)_T^2 \left(\frac{\partial v}{\partial T} \right)_p^2 - \left(\frac{\partial p}{\partial v} \right)_T \right]^{\frac{1}{2}} \tag{7}$$

Since the specific volume and density are inversely proportional, that is $v=1/\rho$, the following relation for the partial derivatives is obtained:

$$\left(\frac{\partial p}{\partial v} \right)_T = \left(\frac{\partial p}{\partial \rho} \right)_T \left(\frac{\partial \rho}{\partial v} \right)_T = -\rho^2 \left(\frac{\partial p}{\partial \rho} \right)_T \tag{a}$$

and

$$\left(\frac{\partial v}{\partial T} \right)_p = \left(\frac{\partial \rho}{\partial T} \right)_p \left(\frac{\partial v}{\partial \rho} \right)_p = -\frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial T} \right)_p \tag{b}$$

When the previous relations are inserted in Eq. (7), relation in the following form is obtained:

$$\frac{N_L}{N_A} = \frac{1}{u \cdot M} \left[\frac{T}{c_v} \left(\frac{\partial p}{\partial \rho} \right)_T^2 \left(\frac{\partial \rho}{\partial T} \right)_p^2 + \rho^2 \left(\frac{\partial p}{\partial \rho} \right)_T \right]^{1/2} \tag{8}$$

The relative deviation of the N_L/N_A ratio from the mean value of real gases: argon, hydrogen, helium, carbon monoxide, carbon dioxide, nitrogen, oxygen, and methane, is calculated as follows:

$$\Delta\left(\frac{N_L}{N_A}\right)=\frac{\left(\frac{N_L}{N_A}\right)_{average}-\left(\frac{N_L}{N_A}\right)_{gas}}{\left(\frac{N_L}{N_A}\right)_{average}}100\% \tag{9}$$

3. NUMERICAL ANALYSIS

3.1. Numerical analysis of the constant ratios and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at standard state (temperature 273.15 K and pressure 101325 Pa)

The calculation of the constant ratios N_L/N_A and the relative deviation of the N_L/N_A ratio for the real gases at standard state (i.e., temperature 273.15 K and pressure 101,325 Pa) show a good agreement with the results in the literature, given the fact that the literature data of the N_L/N_A ratio is 0.04461498 at standard state, Ref. [3, 6]. The mean value of the constant ratios N_L/N_A is 0.04467675. **Table 1** data shows a good agreement with the literature because the absolute difference appears in the fifth decimal and reaches its maximum value for methane, carbon dioxide, nitrogen, and helium. **Table 1** and **Figure 1** present the absolute values and relative deviations from the mean value of the N_L/N_A ratio for the real gases were calculated using their real-equations of state at standard state, according to Ref. [11 to 19], as well as Eqs. (3), (8), (9). Thus, **Table 1** shows that the absolute differences from the mean value of the N_L/N_A ratio range from 0.00002779 for oxygen to 0.00024082 for carbon dioxide, while the relative deviations range from 0.03981548% for oxygen to 0.5390307% for carbon dioxide.

Table 1 The calculations for real-gases using their real-equations of state, Ref. [11 to 19]

Gas	M, kg/kmol	c_v, J/kgK	u, m/s	$(\partial p/\partial \rho)_T$, Pa/(kg/m^3)	$(\partial \rho/\partial T)_p$, kg/m^{3}K	N_L/N_A; Eq. (8)	$\Delta(N_L/N_A)$, %; Eq. (9)
H_2	2.01588	10070.55	1261.06	1128010.0	-0.000328954	0.04458716	0.200521
He	4.00260	3116.176	972.940	568010.6	-0.000653797	0.04455534	0.1914395
CO	28.0101	743.1707	336.942	80974.04	-0.004596181	0.04464465	0.07184297
Ar	39.948	312.4416	307.857	56744.36	-0.00655719	0.04465695	0.04430984
N_2	28.013	742.8848	336.969	81061.654	-0.00460508	0.04471501	0.09137965
O_2	31.9988	655.2213	314.8105	70836.00	-0.005253419	0.04465896	0.03981548
CO_2	44.0098	632.0157	258.0763	50909.44	-0.007399405	0.04491757	-0.5390307
CH_4	16.0428	1656.75	430.667	140890.5	-0.002648078	0.04472155	-0.1002850

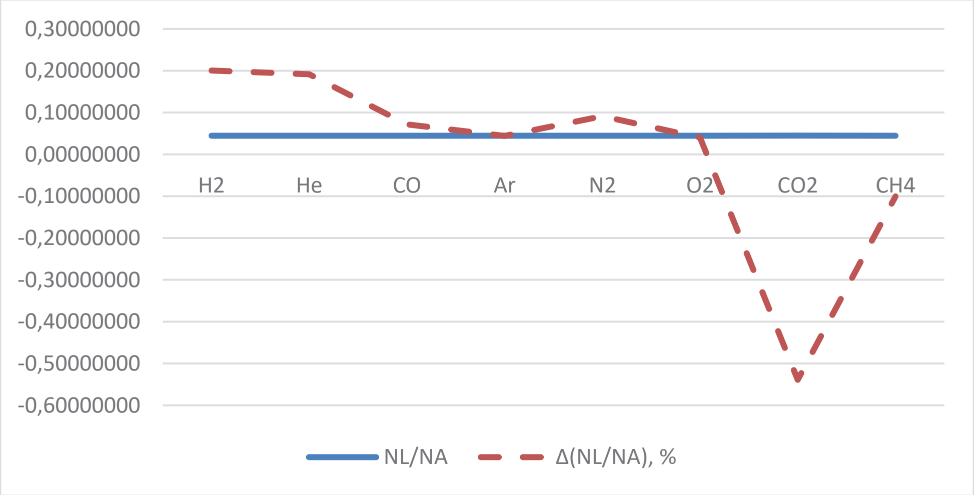

Figure 1 *The ratio of the constants and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at standard state (i.e., temperature 273.15 K and pressure 101325 Pa)*

3.2. Numerical analysis of the constant ratios and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at a pressure of 500,000 Pa and temperature of 293.15 K

The constant ratios N_L/N_A and the relative deviations from the mean value of the N_L/N_A ratio for the real gases at state (temperature 293.15 K and pressure 500,000 Pa) were calculated and presented in **Table 2** and **Figure 2** using their real-equations of state, according to Ref. [11 to 19], as well as Eqs. (3), (8), (9). The mean value constant ratios N_L/N_A is 0.2061974. Thus, **Table 2** shows that the absolute differences from the mean value of the N_L/N_A ratio range from 0.0003472 for oxygen to 0.0045752 for carbon dioxide, while the relative deviations range from 0.1683953% for oxygen to 2.218877% for carbon dioxide.:

Table 2 The calculations for real-gases using their real-equations of state, Ref. [11 to 19]

Gas	M, kg/kmol	c_v, J/kgK	u, m/s	$(\partial p/\partial \rho)_T$, Pa/(kg/m^3)	$(\partial \rho/\partial T)_p$, kg/m^{3}K	N_L/N_A; Eq. (8)	$\Delta(N_L/N_A)$, %; Eq. (9)
H_2	2.01588	10164.56	1308.066	1216278.0	-0.001404064	0.2045311	0.8081208
He	4.00260	3117.16	1009.697	611916.9	-0.002786614	0.2046398	0.7554015
CO	28.0101	744.5997	349.5669	86683.76	-0.01994752	0.2055418	0.3179580
Ar	39.948	313.1745	319.2481	60605.88	-0.02850255	0.20583031	0.1780429
N_2	28.013	744.1177	349.6914	86812.89	-0.01988569	0.205376814	0.3979787
O_2	31.9988	658.75	326.014	75648.23	-0.02284682	0.2058502	0.1683953
CO_2	44.0098	663.3485	263.296	52433.20	-0.03458745	0.21077266	-2.218877
CH_4	16.0428	1701.507	443.6379	149159.1	-0.01171133	0.2061974	-0.4070265

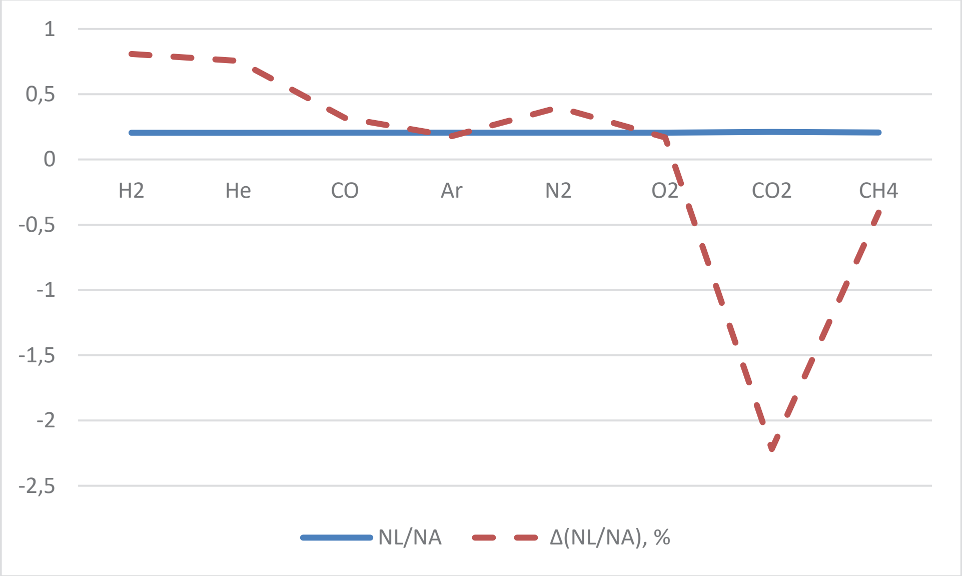

Figure 2 *The ratio of the constants and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at pressure of 500,000 Pa and temperature of 293.15 K*

3.3 Numerical analysis of the constant ratios and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at a pressure of 1000,000 Pa and temperature of 320.0 K

The constant ratios N_L/N_A and the relative deviation from the mean value of the N_L/N_A ratio for the real gases at state (temperature 320.0 K and pressure 1000,000 Pa) were calculated and presented in **Table 3** and **Figure 3** using their real-equations of state, according to Ref. [11 to 19], as well as Eqs. (3), (8), (9). The mean value of the constant ratios N_L/N_A is 0.3784468125. Thus, **Table 3** shows that the absolute differences from the mean value of the N_L/N_A ratio range from 0.0009857 for oxygen to 0.0131547 for carbon dioxide, while the relative deviations range from 0.2604626% for oxygen to 3.475967% for carbon dioxide.

Table 3 The calculations for real-gases using their real-equations of state, Ref.[11 to 19]

Gas	M, kg/kmol	c_v, *J/kgK*	u, m/s	$(\partial p/\partial \rho)_T$, Pa/(kg/m^3)	$(\partial \rho/\partial T)_p$, kg/m^{3}K	N_L/N_A; Eq. (8)	$\Delta(N_L/N_A)$, %; Eq. (9)
H_2	2.01588	10252.78	1368.837	1334725.0	-0.00234579	0.3737498	1.2411288
He	4.00260	3118.185	1056.824	670601.1	-0.004657565	0.3741995	1.1230735
CO	28.0101	746.4248	366.2333	94750.75	-0.0337198	0.376373	0.547927
Ar	39.948	313.7312	334.2333	66055.13	-0.0482924	0.3774277	0.269288
N_2	28.013	745.4526	366.4218	94960.55	-0.03355621	0.375925	0.6663585
O_2	31.9988	663.9546	340.7687	82450.42	-0.03872109	0.3774611	0.2604626
CO_2	44.0098	696.045	271.8047	55632.12	-0.06171518	0.3916015	-3.475967
CH_4	16.0428	1770.058	461.1317	161557.4	-0.02010286	0.3808367	-0.6314989

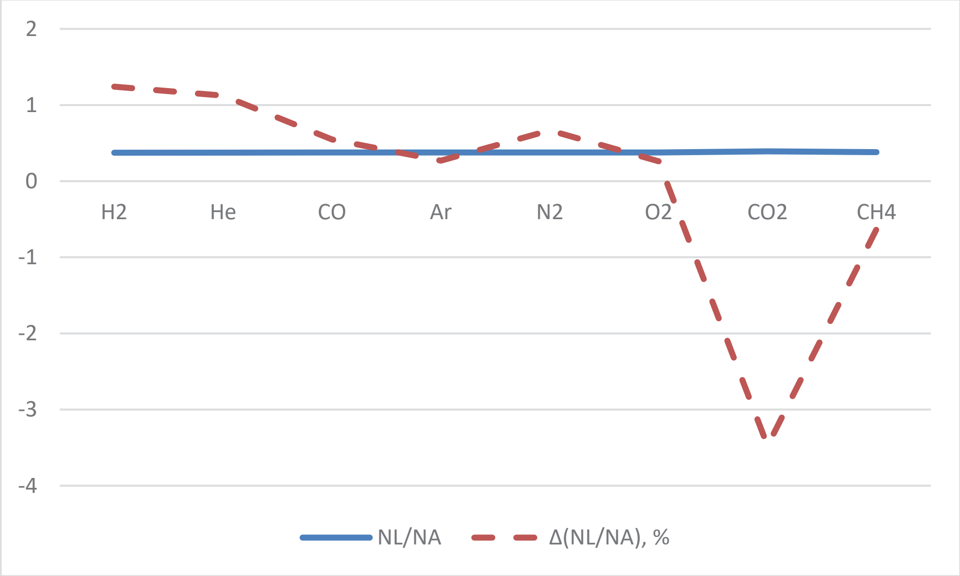

Figure 3 *The ratio of the constants and the relative deviation of the N_L/N_A ratio from the mean value for the real gases at pressure of 1000,000 Pa and temperature of 320.0 K*

4. CONCLUSION

■ The contribution of the paper is reflected in the derivation of equation (8), which defines the analytical relationship between Avogadro and Loschmidt numbers as a function of the speed of sound and *pvT* - the thermodynamic properties of the gas. The Loschmidt constant is approximately proportional to the Avogadro constant, according to Ref. [6], which is also numerically confirmed for real-gases in the paper (argon, hydrogen, helium, carbon monoxide, carbon dioxide, nitrogen, oxygen, and methane).

■ The number of particles (atoms or molecules) in the same volume for all gases is equal at the same temperature and pressure. The conclusion is due to the fact that the N_L/N_A ratio is calculated using equation (8) almost the same for the considered gases, namely the absolute differences appear on the fourth decimal place for carbon dioxide and methan, and the fifth decimal place for the other considered gases (**Table 1**). The conclusion corresponds to the Avogadro's hypothesis 'that equal volumes of all gases at the same temperature and pressure contain the same number of molecules'. Therefore, there are greater absolute differences and relative deviations of the constant ratios from the mean value for much higher *pvT*-thermodynamic gas properties when compared to the standard state, as shown in **Table 2**, **3**.

■ The authors consider this occurs primarily because of the uncertainty of equations of state of real gases used in calculations, in accordance with Ref. [11 to 19]. The heat capacity of gases also appears in equation (8) and, as is known, in real gases the heat capacities depend simultaneously on both pressure and temperature, in contrast to ideal gases where the heat capacities depend only on temperature. Also, in relation (8), the speed of sound is calculated using equation (5) and the corresponding state equations of real gases, although there is also an option for experimental determination. According to the above, the suggestion is that the derived equation (8) can be a good test for the newly derived equations of state for real gases because, simply put, the ratio defined by equation (8) must be the same for all gases under the same *pvT* – the thermodynamic properties of the gases.

■ The velocities of sound are different in gases under the same *pvT* – the thermodynamic properties of the gas, despite the fact that in these circumstances all gases have the same number of particles in the same volumes (Avogadro's hypothesis). Thus, it is clear that the speed of sound in gases depends on the type of gas (i.e. molecular weight and heat capacity of the gas) and temperature. In other words, all gases under the same *pvT* – the thermodynamic properties of the gas have the same number of particles (atoms or molecules) in equal volumes, but in these circumstances have different the speed of sound.

■ The explanation of the previous conclusion is possible by analogy with the conduction of thermal energy in gases. Sound is an energy form, as is thermal energy. Thus, the propagation of sound in gases can be compared to the propagation of thermal energy. In the theory of heat transfer in gases and liquids, there are very successful correlations between the coefficient of heat conduction and the speed of sound in these fluids. The heat conduction coefficient of gases can be observed to be several times higher for hydrogen and helium, compared to the coefficients of other gases, what is also the same with the speed of sound. This fact is explained by that hydrogen and helium have small molecular masses compared to other gases and therefore have a higher mean velocity of elementary particles in volume, which is directly proportional in functional relations to both the coefficient of heat conduction and the speed of sound in gas. Thus, the mean velocity of elementary particles in the volume of a gas directly and in the same way affects both the conduction of thermal energy and the propagation of sound.

■ Experimental, as well as calculated, data indicate the fact that the speed of sound in real gases increases with increasing pressure, i.e., gas density (**Table 1**, **2**, **3**). On the other hand, according to the kinetic theory of gases, increasing the pressure (i.e. density) of a gas reduces the mean trajectory of a particle (atom, molecule) during the interval between two collisions. This indicates that the increase in gas pressure (density) will not have as significant an impact on the increase in the speed of sound in the gas as the type and temperature of the gas, because in functional relations the density and mean particle path between two collisions are proportional to both heat conduction coefficient and speed of sound.

■ Can it be said that matter (i.e. atoms or molecules) is also an energy form? Namely, in one of Nikola Tesla's last interviews, to the journalist's remark 'that his claim that matter has no energy – is very strange', Tesla replied: 'first it was energy, then matter'. Explaining the birth of the universe, Tesla says 'Matter is created from the original and eternal energy we know as light. Matter is an expression of infinite forms of light, so energy is older than it' - also an interesting statement. On the other hand, from the well-known Albert Einstain's equation ($E=mc^2$) it follows that energy is proportional to mass with a constant of proportionality - the speed of light in a vacuum squared. Is that why equation (8), but also numerous other equations in thermodynamics, necessarily contain atomic or molecular mass and heat capacities of gases. Sound in a given volume of gas (i.e., system) is the result of bringing energy from another system (for example environment) into a system where sound propagation occurs. Thus, one part of the energy supplied to the system (i.e., gas) is transformed into sound energy and the other part into other energy forms in the system. According to the First and the Second Law of Thermodynamics, a very complex interaction of different energy forms and simultaneous transformation of one energy form into another occurs in the considered volume of gas (i.e., system) during the process of sound propagation in gas, for example: thermal, mechanical, internal (translational, rotational, and vibrational energy of particles), kinetic, potential, sound energies, etc.

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