PRIMJENA METODA *MONTE CARLO* ZA OPTIMIZACIJU MATEMATIČKOG MODELA HRAPAVOSTI POVRŠINE

APPLICATION OF THE MONTE CARLO METHOD FOR THE OPTIMIZATION OF MATHEMATICAL MODEL OF SURFACE ROUGHNESS

Sanel Gredelj[1]
Sanela Hrnjica[1]

[1]*University od Bihać Faculty of Technical Engineering*

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REZIME
U ovom radu pokazana je optimizacija primjenom metoda Monte Carlo*, matematičkog modela hrapavosti, odnosno srednjeg aritmetičkog odstupanja profila. Optimizacija je pronalaženje najboljeg od svih mogućih rješenja. Prethodno je izvršen eksperiment, prema planu eksperimenta. Na osnovu rezultata eksperimenta i plan-matrice određeni su koeficijenti matematičkog modela. Adekvatan i nelinearan matematički model, pogodan za optimizaciju, daje zavisnost srednjeg aritmetičkog odstupanja profila od broja obrtaja, posmaka, promjera alata i dubine rezanja. Metod* Monte Carlo *simulira neku pojavu ili proces izvođenjem brojnih fiktivnih eksperimenata pomoću slučajnih brojeva. Primjena metoda* Monte Carlo *u optimizaciju nije originalna. Međutim, prikazana tehnika u ovom radu, korištenjem softvera* MS Excel*, jeste originalna, pouzdana i jednostavna. Nedostatak je nepreciznost. Preporučuje se da se ovaj metod za optimizaciju koristi kao kontrolni metod optimizacije, u odnosu na neki drugi metod.*

Professional paper

SUMMARY
This paper shows optimization by using the Monte Carlo method, the mathematical model of roughness, or the mean arithmetic deviation of the profile. Optimization means finding the best of all possible solutions. According to the experiment plan, an experiment was previously performed. Based on the results of the experiment and the plan matrix, the coefficients of the mathematical model were determined. An adequate and nonlinear mathematical model, suitable for optimization, gives the dependence of the mean arithmetic deviation of the profile from the number of main spindle revolutions, feed rate, tool diameter and cutting depth. The Monte Carlo method simulates action or proces by performing number of fictitious experiments using random numbers. The application of Monte Carlo method in optimization is not original. However, using MS Excel as a technique shown in this paper is original, reliable and simple. The disadvantage is imprecision. It is recommended to use Monte Carlo method as a control method in relation to some other method.

1. INTRODUCTION

Roughness of surface is an essential characteristic of machine elements and has an impact on the properties of machine parts and structures, especially in places of mutual connection. Surface quality affects the dynamic durability of the element. Therefore, it is important to ensure as little roughness as possible, because in addition to functional, it also has an aesthetic role, especially in aluminum elements. During the part production on CNC machines, there are numerous parameters that affect the quality of the surface, in this paper four input parameters have been taken in account on five levels: the number of main spindle revolutions *n*, feed rate *s*, tool diameter *d* and cut depth *a*. The output of the process is mean arithmetic deviation of the profile *Ra*. During the experiment, the parameters were varied according to the plan of the experiment, within the limits which are applicable in production. Based on the results of the experiment and the plan matrix, the coefficients of the mathematical model were determined. Optimization means finding the best solution of all possible solutions. The result of optimization are those parameter values that give the optimal value of roughness.

Optimization is done by using the Monte Carlo method, that simulates a phenomenon or process by performing an experiment of random numbers. A simulation is the execution of a model, represented by a computer program that gives information about the system being investigated. Monte Carlo Simulation, also known as the Monte Carlo Method or a multiple probability simulation, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. It was invented during the Manhattan Project by John von Neumann [1903-1957], Nicholas Metropolis [1915-1999] and Stanislaw Ulam [1909-1984] and named for Ulam's uncle, who enjoyed playing games of chance in Monte Carlo, Monaco. This is a probabilistic method based on performing numerous fictive experiments using random numbers. The Monte Carlo method requires the use of a computer because a large number of random variables need to be generated [1, 3, 6, 8, 9].

In mathematics, engineering, computer science and economics, an optimization problem is the problem of finding the best solution from all feasible solutions. There are many Mathematical Programming Techniques, Stochastic (Probabilistic) Process Techniques and Statistical Methods to solve optimization problems. Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic or Probabilistic process techniques can be used to analyze problems described by a set of random variables having known probability distributions. Statistical methods enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation [2, 4, 5,7].

In this paper the Monte Carlo method is introduced for optimizing mathematical models of machining processes. This technique is original and simple and uses MS Excel software.

2. RANDOM NUMBER, RANDOM VARIABLE AND STOCHASTIC PROCESS GENERATION

Most of today's random number generators are not based on physical devices but on simple algorithms that can be easily implemented on a computer. These generators are called pseudo-random. Pseudo-random number generators require tests as exclusive verifications for their "randomness," as they are decidedly not produced by "truly random" processes, but rather by deterministic algorithms. Over the history of random number generation, many sources of numbers thought to appear "random" under testing have later been discovered to be very non-random when subjected to certain types of tests. The notion of pseudo-random numbers was developed to circumvent some of these problems, though pseudo-random number generators are still extensively used in many applications as they are good enough for most applications. Non-parametric hypothesis tests can be used for testing randomness of sequence random numbers. There are several popular types of non-parametric hypothesis tests used in research nowadays: chi-square χ^2 test, Friedman one, Romanovsky, Kolmogorov-Smirnov one and others [3, 10].

It is easy to generate uniformly distributed pseudo-random numbers on a computer. The probability that a uniformly distributed random number falls within any interval of fixed length is independent of the location of the interval itself (but it is dependent on the interval size), so long as the interval is contained in the distribution's support.

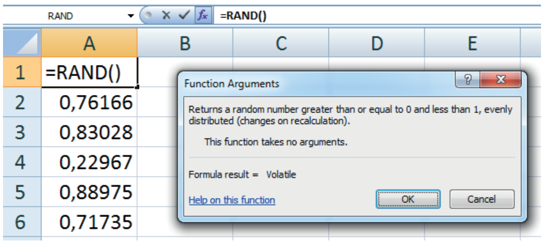

Figure 1 *RAND() function in MS Excel*

In MS Excel, the RAND () function is used to generate random numbers in the interval between 0 and 1 (Figure 1).

Random numbers, distributed uniformly in the interval [0;1], are the basis for determining random variables. The basic idea is that the cumulative distribution function F for any continuous random quantity x is also random number. If the cumulative distribution function of random quantity x is described by the expression [1]:

$$u = F(x) \quad (1)$$

then the random variables of x can be obtained from the random numbers u with uniform distribution in (0; 1) using the inverse formula:

$$x = F^{-1}(u) \quad (2)$$

Here, F^{-1} means inverse probabilistic transformation (Figure 2).

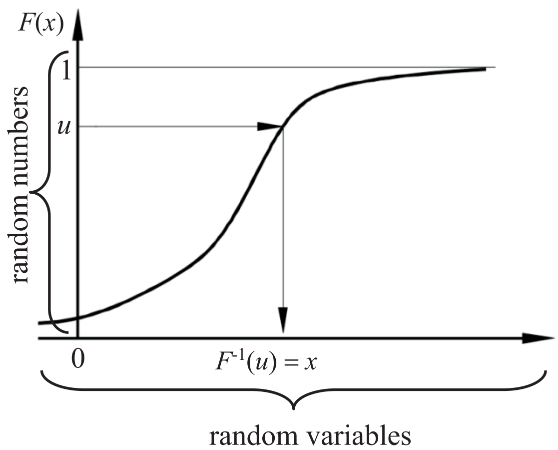

Figure 2 *Generation of random variables by inverse probabilistic transformation*

For example, the cumulative distribution function for normal distribution is:

$$u = F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (3)$$

with the parameters μ and σ. The inverse transformation for the distribution (3) does not have analytical solutions.

The value of random variable is obtained from function NORMINV (A2;F1;F2), which is inverse function of normal distribution, and random number, function RAND () (Figure 3). Specific parameter values are shown in cells F1 and F2. In this case the probability is represented by pseudo-random number =RAND().

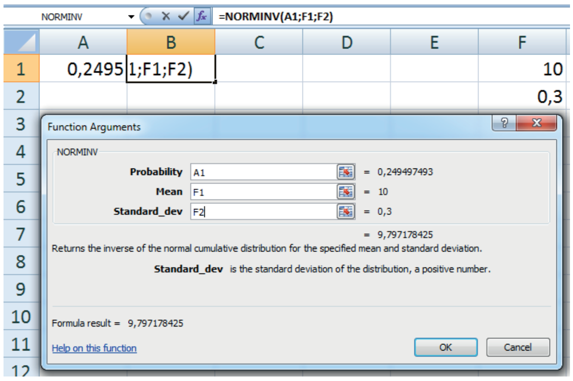

Figure 3 *Generation of random variables using MC Excel*

The simulation of a random process is obtained by copying the first row in sufficiently large number of iterations.

If the function F can be distributed uniformly in the interval $[a,b]$, then the calculation of random value for variable $x_i \in [a,b]$ can be done based on equation [4]:

$$rand(x_i) = (b-a) \cdot RAND() + a \quad (4)$$

which is the basis for the Monte Carlo method.

3. MATHEMATICAL MODEL OPTIMIZATION BY MONTE CARLO METHOD OF SIMULATION

Mathematical model optimization by Monte Carlo method is explained and shown using the example of optimization for non-linear coded mathematical model of arithmetical profile surface deviation:

$$Ra = 0{,}1999 - 0{,}4473 \cdot X_1 - 0{,}1056 \cdot X_2 +$$

$$
\begin{aligned}
&+0{,}2151 \cdot X_3 + 0{,}0406 \cdot X_4 + 0{,}4972 \cdot X_1^2 + \\
&+0{,}2234 \cdot X_2^2 + 0{,}2141 \cdot X_3^2 + 0{,}2221 \cdot X_4^2 + \quad (5) \\
&+0{,}0519 \cdot X_1 X_2 - 0{,}066 \cdot X_1 X_3 - \\
&-0{,}0576 \cdot X_1 X_4 - 0{,}0313 \cdot X_2 X_3 - \\
&-0{,}0021 \cdot X_2 X_4 - 0{,}0141 \cdot X_3 X_4
\end{aligned}
$$

Coded values are:

$$
\left.
\begin{aligned}
X_1 &= 2\frac{d-d_0}{d_{+1}-d_{-1}} = 2\frac{d-10}{12-8} = \frac{d-10}{2} \\
X_2 &= 2\frac{n-n_0}{n_{+1}-n_{-1}} = 2\frac{n-8000}{8500-7500} = \frac{n-8000}{500} \\
X_3 &= 2\frac{s-s_0}{s_{+1}-s_{-1}} = 2\frac{s-3000}{3500-2500} = \frac{s-3000}{500} \\
X_4 &= 2\frac{a-a_0}{a_{+1}-a_{-1}} = 2\frac{a-0{,}6}{0{,}7-0{,}5} = \frac{a-0{,}6}{0{,}1}
\end{aligned}
\right\} \quad (6)
$$

and the parameters are:

- d [mm] tool diameter,
- n [rpm] number of main spindle revolutions,
- s [mm/min] feedrate,
- a [mm] cutting depth.

From equations (6) follows:

$$
\left.
\begin{aligned}
d &= 2X_1 + 10 \\
n &= 500X_2 + 8000 \\
s &= 500X_3 + 3000 \\
a &= 0{,}1X_4 + 0{,}6
\end{aligned}
\right\} \quad (7)
$$

Optimization is done by using MS Excel, and shown by steps, the goal was to get minimal value of *Ra* defined by specific values of parameters.

1. The random values of input parameters are generated by equation (4) and limits $-2 \le X_i \le 2$, $i = 1,2,3,4$.

B2 =(2-(-2))*RAND()+(-2)

	A	B	C	D	E
1	*N*	*X* 1	*X* 2	*X* 3	*X* 4
2	1	1,33028	-1,17954	-0,40138	-1,33646

Figure 4 *Step 1 – Random values generated*

2. The given mathematical model equation is defined in corresponding cell (E2), where the cell addresses are used instead of input variables: B2 = X_1, C2 = X_2, D2 = X_3 and E2 = X_4.

F2 =0,199-0,4473*B2-0,1056*C2+0,2151*D2+0,0406*E2+0,0519*B2*C2-0,066*B2*D2-0,0576*B2*E2-0,0313*C2*D2-0,0021*C2*E2-0,0141*D2*E2+0,4972*B2*B2*
0,2234*C2*C2+0,2141*D2*D2+0,2221*E2*E2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	*N*	*X* 1	*X* 2	*X* 3	*X* 4	*Ra*									
2	1	-0,30452	0,922381	1,703088	0,942597	1,432064									

Figure 5 *Step 2 – Mathematical model equation*

3. Row 2 is copied arbitrary number of times, to get sufficient number of iterations. In this case that number is 50.000.

A50001 =A50000+1

	A	B	C	D	E	F
49998	49997	-0,26464	-1,12583	-0,4485	1,200622	1,091366
49999	49998	0,779538	-1,43322	-1,54421	-0,35526	0,914701
50000	49999	1,768301	0,651285	-0,87598	-1,99953	2,133816
50001	50000	1,408976	1,165786	1,511412	-1,8868	2,352747

Figure 6 *Step 3 – Copying until sufficient iteration number*

4. The minimal value is defined in column F, for randomly determined values of *Ra*.

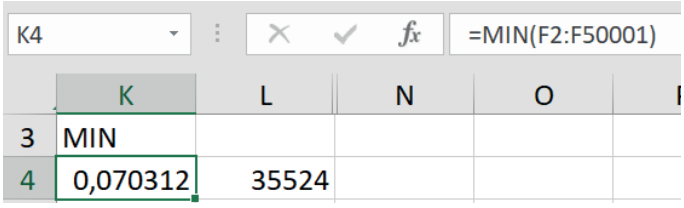

Figure 7 *Step 4 – Determination of minimal value*

5. Number of rows for column F, in which is the minimal value, are determined by using the function =MATCH (K4; F: 50001; -1). The -1 is written because in this case the first row is not counted.

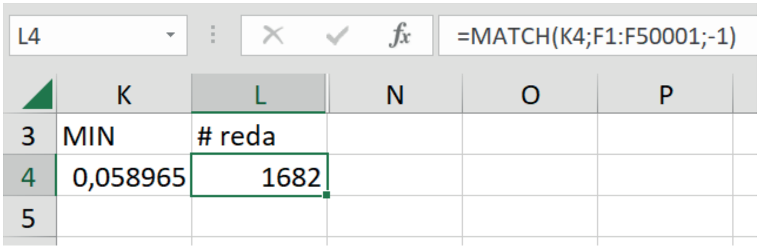

Figure 8 *Step 5 – Determination of the row number where is the minimal value*

6. The value of input parameter X_1 is shown by function =INDIRECT("B"&L4), for certain row in column F where is the minimal value, and in the combination with other input parameters, they give minimal value of *Ramin*.
In the similar way by using =INDIRECT("C"&L4), =INDIRECT("D"&L4) and =INDIRECT("E"&L4), the requested values of input parameters are given *X2, X3,* and *X4* respectively.
By decoding based on equations (6) and (7), the results are values of input quantities for minimal value of *Ra*, (Figure 9).

O10 =INDIRECT("B"&L4)

	L	N	O	P	Q	R	S	T	U
10			0,3377	X 1	10,675	d	8	12	21,351
11			0,1729	X 2	8086,5	n	7500	8500	16173
12	Ramin	0,0605	-0,507	X 3	2746,6	s	2500	3500	5493,1
13			-0,203	X 4	0,5797	a	0,5	0,7	1,1594

Figure 9 *Step 6 – Defined input parameters for minimal value*

The exact values can be shown by using another optimization methods, with input parameter values of:

$d_{opt} = 10{,}8421$
$n_{opt} = 8087$
$s_{opt} = 2797$
$a_{opt} = 0{,}05975$, (Figure 9).

4. CONCLUSION

Optimization of mathematical models with the Monte Carlo method is simple and reliable process. By using the widely available MS Excel program, which is user-friendly because of its accesible interface, mistakes are immediately shown, without mistrust of background calculations and possible code errors.
Method is based on typical (generic) behavior of processes, because the input values are realistic in comparison with mathematical model, that means there are no unrealistic theoretical values.
The Monte Carlo simulation method is a numerical stochastic model, which requires a large iteration number. Although the iteration number is arbitrary, for optimization application it should be 10^4 order. Monte Carlo optimization immediately determines the kind of extremes in the process, whether it is a maximum or a minimum. Interval values of input variables do not have any impact on method efficiency. Also, there are no additional constrains for mathematical model, such as

complexity, continuity, linearity and differentiability, it is totally arbitrary.
Disadvantage of Monte Carlo optimization is inaccuracy which can be eliminated by increased number of iterations.
But regardless of some disadvantages, the Monte Carlo method is suitable, due to its simplicity and can be used as a control method, compared to optimization with a genetic algorithm, neural networks or classical method.
If it is found that the optimal values, obtained by Monte Carlo optimization, are similar to the values obtained by some other optimization method, there is almost no doubt that the results are correct. There is no theoretical possibility that the results are wrong if the both methods give similar results.

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Corresponding author:
Sanel Gredelj
University of Bihać
Faculty of Technical Engineering
I. Ljubijankića bb, Bihać
Email: sanel.gredelj@unbi.ba
Phone: +387 61 464 246