

KOMPARACIJA ANALITIČKOG I EKSPERIMENTALNOG ODREĐIVANJA I OPTIMIZACIJA SILE SAVIJANJA

COMPARISON OF ANALYTICAL AND EXPERIMENTAL DETERMINATION AND OPTIMIZATION OF BENDING FORCE

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REZIME

Analitička rješenja nekog problema su kombinacija prirodnih zakona, matematičkih pravila i aproksimacija. Jedini način da se ustanovi da li analitička rješenja odgovaraju realnosti je eksperimentalna verifikacija. U ovom radu su analizirana aproksimativna analitička rješenja sile čisto plastičnog savijanja sa i bez očvršćavanja materijala zbog deformacije. Eksperiment je izveden u proizvodnim uslovima prema pravilima plana eksperimenta, gdje su ulazni parametri debljina i širina lima, a izlazni sila savijanja. Ustanovljeno je da su eksperimentalni rezultati bliski analitičkom rješenju bez očvršćavanja materijala, odnosno da je očvršćavanje materijala zanemarivo. Na osnovu plana eksperimenta dobijen je adekvatan nelinearni matematički model, pogodan za optimizaciju. Dobijena optimalna rješenja daju ulazne parametre za minimalnu i maksimalnu silu savijanja. Ustanovljeno je da se maksimalna i minimalna sila savijanja dobiju za približno istu debljinu, ali za različite širine lima.

Professional paper

SUMMARY

Analytical solutions to a problem are a combination of natural laws, mathematical rules and approximations. The only way to determine whether analytical solutions correspond to reality is experimental verification. In this paper, approximate analytical solutions of the purely plastic bending force with and without strain hardening of the material are analyzed. The experiment was performed under production conditions according to the rules of the experimental plan, where the input parameters are the thickness and width of the sheet, and the output is the bending force. It was found that the experimental results are close to the analytical solution without strain hardening of the material, i.e. that the hardening of the material is negligible. Based on the experimental plan, an adequate nonlinear mathematical model was obtained, suitable for optimization. The obtained optimal solutions provide the input parameters for the minimum and maximum bending force. It was found that the maximum and minimum bending force are obtained for approximately the same thickness, but for different sheet widths.

1. INTRODUCTION

Bending is one of the most common methods of processing by deformation, and the application of bending is very wide, from the automotive, food, aircraft industry, shipbuilding etc. With this processing procedure, it is possible to bend sheets, pipes and other profiles of different

thicknesses and materials. Bending is performed on CNC press where, after entering data via the control panel, the appropriate tool is placed on the machine according to the dimensions and material of the workpiece, which makes the processing efficient and optimal [1].

In metal bending processing, the analysis of bending moment and force, which act on the tool and workpiece, and also on the complete structure of the bending press, is of particular interest. Depending on the type and value of the stresses that occur during metal bending, as well as the value of the reduced bend radius, the bending problem is treated in two ways: pure plastic bending and bending in the elastic-plastic region. In purely plastic bending, permanent plastic deformation occurs in the end fibers. When bending in the elastic-plastic region, elastic straightening occurs. Elastic straightening is a negative phenomenon in bending processing, so it is necessary to strive for purely plastic bending, the result of which corresponds to the required specifications of the bent metal.

In this paper, the purely plastic free bending of the V profile is analyzed, where the workpiece is a galvanized steel sheet (DX51DZ Z275). At the same time, an experiment was made according to the appropriate plan with the main goal of comparing the values of the forces obtained analytically and experimentally. In this case, the input variables are the thickness and width of the sheet, as the only geometric quantities that in the analytical forms have an influence on the values of the bending moments. The second goal of the experiment is to obtain an adequate nonlinear mathematical model on the basis of which the bending force can be optimized.

Since it is a practical, industrial application, it is impossible to list all the literature that dealt with this problem. However, books [1-4] can be singled out as relevant, in which bending is analyzed as one of the plastic deformation processing processes, as well as the associated geometry, moments and forces. Bending of V profiles, analytical and experimental analysis, has also been addressed in many works from which they can be singled out [5-8].

In the paper [8] it was established that the bending force is not affected by the bending angle, that the influence of the bending length is insignificant, and that the sheet thickness has the greatest influence, while the width of the sheet was not considered.

2. GEOMETRY, MOMENTS AND BENDING FORCE OF V PROFILES

The main advantage of free bending of the V profile is that the machines and tools are of a universal character. One tool can bend sheets at

various angles, of different materials and geometric properties. This results in the disadvantage that free bending is applied for the production of a smaller number of pieces in individual and small-batch production [2].

In order to determine whether the bending process of the V profile is in the purely plastic or in the elastic-plastic region, it is necessary to calculate the reduced radius of curvature of the neutral line [4]:

$$\rho_r = \frac{\rho_n}{s} = \sqrt{\frac{r}{s} \left(1 + \frac{r}{s} \right)}, \quad (1)$$

where, figure 1:

ρ_n – radius of curvature of the neutral line,

r – radius of the upper tool,

s – thickness of the sheet.

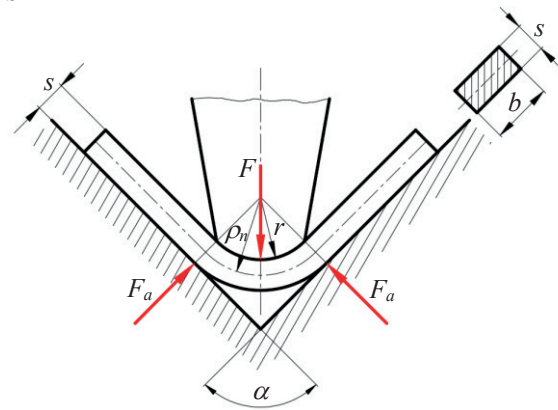


Figure 1 Bending of V profiles

For $5 < \rho_r < 200$ the bending process is in the elastic-plastic region, for $\rho_r < 5$ it is a purely plastic bending, while for $\rho_r > 200$ it is an elastic deformation and the sheet is straightened to its original state after the application of force. The bending moment in the purely plastic region is calculated according to the formula [2,9]:

$$M_v = \beta \cdot \sigma_v \cdot \frac{b \cdot s^2}{4}, \quad (2)$$

where:

σ_v – yield strength of the material, which for galvanized steel is $\sigma_v = 0,235$ [kN/mm²],

β – Lode's coefficient, which value is

$$1 \leq \beta \leq \frac{2}{\sqrt{3}}.$$

If the influence of material hardening during cold processing is taken into account, according to V. Romanovski, a simplified form is used [4]:

$$Mm = n \cdot \sigma_m \cdot \frac{b \cdot s^2}{4}, \quad (3)$$

where:

n – correction factor with values $1,6 \leq n \leq 1,8$,
 σ_m – ultimate tensile strength of the material, which for galvanized steel is $\sigma_m = 0,45$ [kN/mm²].

The bending force of V profile is calculated according to the general expression:

$$F = \frac{2M}{r + 0,5s} \operatorname{ctg} \frac{180^\circ - \alpha}{2}, \quad (4)$$

where α is the bending angle, figure 1.

By inserting formulas (2) and (3) into formula (4), the formulas for bending forces in the purely plastic region are obtained:

$$Fv = 2 \cdot \beta \cdot \sigma_v \cdot \frac{b \cdot s^2}{4} \cdot \frac{1}{r + 0,5s} \operatorname{ctg} \frac{180^\circ - \alpha}{2}, \quad (5)$$

$$Fm = 2 \cdot n \cdot \sigma_m \cdot \frac{b \cdot s^2}{4} \cdot \frac{1}{r + 0,5s} \operatorname{ctg} \frac{180^\circ - \alpha}{2}, \quad (6)$$

with formula (6) taking into account the hardening of the material in the cold state.

3. RESEARCH EQUIPMENT

For the experiment, the CNC press “Amada HFE 3i 1703L” (figure 2) was used, with its technical details listed in table 1.

The sheet metal bending process involves two main components: the upper and lower tools, which are mounted on the machine as seen in figure 3. The tools are securely attached to the machine’s supports — the upper tool is fixed in place by rotating a handle, while the lower tool is secured with screws. The upper tool, or punch, carries out the bending action, applying force to the workpiece, while the lower tool, known as the die, stays stationary. Both tools are segmented, which allows for customization based on the required bending length and the geometry of the workpiece. Table 2 provides the details of these tools.



Figure 2 CNC press brake “Amada HFE-1703L”

Table 1 Characteristics of the CNC press

Characteristic	Unit	Value
Press capacity	kN	1700
Press beam length	mm	3170
Distance between frames	mm	2700
Stroke	mm	200 (350)
Open height	mm	470 (620)
Bending speed	mm/s	10
Approach speed	mm/s	100
Return speed	mm/s	100
Length	mm	4240
Width	mm	2650
Height	mm	3140
Weight	kg	12710
Press beam length	mm	3170
Machine brand	-	HFE-1703L

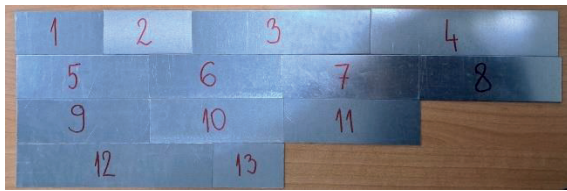


Figure 3 Bending tool parts

Table 2 Characteristics of the tool used for sheet metal bending

Characteristic	Unit	Value
Manufacturer	-	“Amada”
Tool material	-	42CrMo4
Punch radius	mm	0,60-0,65
Punch angle	°	30-88
Maximum force of the upper tool	kN/m	1000
Maximum force of the lower tool	kN/m	400-600

The workpiece consists of a galvanized steel sheet with a fixed length of 50 mm, while the width varies according to the design of the experiment matrix. The samples used in the experiment were laser cut from a metal plate to the specified dimensions, as shown in figure 4. The details of the workpiece are provided in table 3.

**Figure 4** Workpiece used in the experiment**Table 3** Characteristics of the workpiece

Characteristic	Unit	Value
Material – galvanized steel	-	
Length	mm	80-221
Width	mm	50
Thickness	mm	0,7-1,7
Density	kg/m ³	7850
Tensile strength	MPa	450
Yield strength	Mpa	235

Since a CNC press brake is used, the machine automatically measures the bending force for each working stroke and displays it on the control panel. This bending force is generated by two cylinders, which transfer the load through the upper tool (punch) to the workpiece, which rests on the lower tool (die). The force shown on the display is updated in real-time, allowing easy tracking of its changes from the moment the punch first contacts the

workpiece until the bending process is fully completed and the load is released. The machine's display showing this force is illustrated in figure 5.

**Figure 5** Display of the control panel

The total value of the achieved force F_E is presented as sum of the values of the individual forces on first F_{E1} and the second F_{E2} cylinder, i.e. $F_E = F_{E1} + F_{E2}$. Also, changes of these two individual forces on both cylinders are measured, updated and showed on the display of the control panel.

The total force, F_E , is the sum of the individual forces exerted by the first cylinder (F_{E1}) and the second cylinder (F_{E2}), expressed as $F_E = F_{E1} + F_{E2}$. The variations in these two forces are continuously measured, updated, and displayed on the control panel for easy monitoring.

4. EXPERIMENTAL PLAN AND RESULTS

The main goal of this experiment is comparing the forces values obtained analytically and experimentally.

The second goal of this experiment is to output a non-linear mathematical model, so a rotatable plan is used as a special form of the central compositional plan. Rotatable plans have

properties of applicability and optimality, so they are suitable for modeling manufacturing processes that need to be optimized. The rotatable plan contains the base part of the plan $2k$, where k is number of independently variable factors of symmetrically placed point n_α around the center of the plan and the repetition point n_0 in the center of the plan [10,11].

The input parameters of the experiment are: the sheet thickness s [mm] and the width thickness b [mm], so $k = 2$. The output parameter is the bending force F_E [kN].

The required number of measurements is calculated according to the formula [12]:

$$N = 2^k + 2k + n_0 = 2^2 + 2 \cdot 2 + 5 = 13. \quad (7)$$

Coding is performed according to the formula [13]:

$$X_i = \frac{x_i - x_{0i}}{\Delta x_i}, \quad i = 1, 2, 3, \quad (8)$$

where the mean level is the value of the i -th factor:

$$x_{0i} = \frac{x_{i\max} + x_{i\min}}{2}, \quad (9)$$

and variation interval:

$$\Delta x_i = \frac{x_{i\max} - x_{i\min}}{2}. \quad (10)$$

After coding the independently variable values, table 4. (basic factors), the plan matrix of the experiment with the results has the form shown in table 5.

Table 4 Physical and coded values

Influential sizes	Coded and physical values of input factors				
$x_1 = s$ [mm]	0,7	0,9	1,1	1,5	1,7
$x_2 = b$ [mm]	80	100	150	200	221
$X_i, i=1,2$	$-\sqrt{2}$	-1	0	+1	$+\sqrt{2}$

Table 5 Design matrix and results of the experiments

N_j	$X_1 (s)$	$X_2 (b)$	F_E [kN]
1	-1	-1	9,10
2	+1	-1	9,40
3	-1	+1	15,10
4	+1	+1	16,50
5	0	0	17,50
6	0	0	17,80
7	0	0	17,20
8	0	0	17,20
9	0	0	17,50
10	$-\sqrt{2}$	0	7,10
11	$+\sqrt{2}$	0	26,90
12	0	$-\sqrt{2}$	9,40
13	0	$+\sqrt{2}$	24,90

If the input values from the experimental plan are inserted into formulas (5) and (6), with $r = 0,6$ [mm] for $s = 1,5$ [mm] and $s = 1,7$ [mm] (larger thicknesses), while for other thicknesses $r = 0,65$ [mm], the values of the forces shown in table 6 are obtained. The other values are:

$$\sigma_v = 0,235 \text{ [kN/mm}^2\text{]}, \quad \beta = \frac{2}{\sqrt{3}}, \quad \sigma_m = 0,45$$

[kN/mm²], $n = 1,65$. By inserting the values of r and s into equation (1), the values of the reduced radius are obtained.

Table 6 Experimental and analytical values of bending force [kN]

N_j	F_E	F_v	F_m	ρ_r
1	9,10	9,990	26,51	1,56
2	9,40	22,61	60	1,34
3	15,10	19,98	53,02	1,56
4	16,50	43,61	115,71	1,36
5	17,50	20,52	54,45	1,48
6	17,80	20,52	54,45	1,48
7	17,20	20,52	54,45	1,48
8	17,20	20,52	54,45	1,48
9	17,50	20,52	54,45	1,48
10	7,10	9,97	26,46	1,69
11	26,90	40,56	107,63	1,31
12	9,40	10,94	29,04	1,48
13	24,90	30,23	80,22	1,48

According to the data in table 6, the values of the reduced radius for all experiments are

$\rho_r < 2 < 5$, which means that the process is deep in the region of plastic bending.

To determine the deviation of the analytical from the experimental values of the bending force, the formula for calculating the relative error in percentage is used, table 7:

$$\Delta F_{Aj} = \frac{F_{Aj} - F_{Ej}}{F_{Ej}}, j = 1, 2, \dots, 13, \quad (11)$$

where F_{Aj} are the analytical values of forces F_{vj} and F_{mj} and F_{Ej} are the experimental values at individual points of the experiment.

Table 7 Relative errors of experimental and analytical values of the bending force

N_j	ΔF_v %	ΔF_m %
1	9,79	191,31
2	140,56	538,30
3	32,33	251,11
4	164,31	601,30
5	17,26	211,14
6	15,29	205,90
7	19,31	216,57
8	19,31	216,57
9	17,26	211,14
10	40,45	272,68
11	50,79	300,10
12	16,43	208,94
13	21,42	222,18
Average	43,42	280,56

The average error for F_v is 6,5 times smaller than the average error for F_m .

It is obvious that the errors are much smaller for forces for which the material hardening is not taken into account. Therefore, in sheet metal bending processes, no significant hardening occurs due to deformation, which can be explained by the small value of the amount of material that is deformed, and whose hardening, if any, is negligibly small.

A graphical representation of the values of bending forces from table 6 is shown in figure 6. From the graph, figure 6, it is obvious that the difference between the forces F_v and F_E is small for small values of the sheet thickness s .

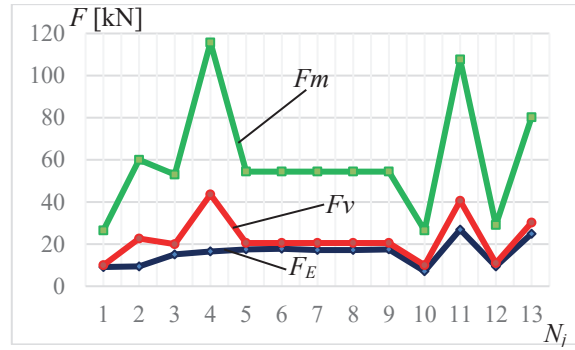


Figure 6 Bending force graph

5. MATHEMATICAL MODEL AND OPTIMIZATION

Using the Design Expert 13 software, based on the experimental plan and results, table 5, a nonlinear mathematical model was obtained, with only significant coefficients:

$$F_R = 17,44 + 7 \cdot X_1 + 5,48 \cdot X_2 + 0,275 \cdot X_1 \cdot X_2 - 2,205 \cdot X_1^2 \cdot X_2 - 6,575 \cdot X_1 \cdot X_2^2 - 4,55 \cdot X_1^2 \cdot X_2^2. \quad (12)$$

The values of the mathematical model for individual experimental points are shown in table 8.

Table 8 Values of experimental and computational results

N_j	X_1	X_2	F_E [kN]	F_R [kN]
1	-1	-1	9,10	9,46
2	+1	-1	9,40	9,77
3	-1	+1	15,10	15,47
4	+1	+1	16,50	15,77
5	0	0	17,50	17,44
6	0	0	17,80	17,44
7	0	0	17,20	17,44
8	0	0	17,20	17,44
9	0	0	17,50	17,44
10	$-\sqrt{2}$	0	7,10	7,54
11	$+\sqrt{2}$	0	26,90	27,34
12	0	$-\sqrt{2}$	9,40	9,69
13	0	$+\sqrt{2}$	24,90	25,19

The regression coefficient is used to examine the agreement (fitting) between F_E and F_R (table 9.) and is calculated according to the formula [10]:

$$R = \sqrt{1 - \frac{\sum_{j=1}^N (F_{Ej} - F_{Rj})^2}{\sum_{j=1}^N (F_{Ej} - \bar{F}_E)^2}}, \quad (13)$$

where there is $\bar{F}_E = \frac{\sum_{j=1}^N F_{Ej}}{13} = 15,73$,

arithmetic mean of all experimental results. According to the value of the experimental and computational results (table 8) it is obtained:

$$R = \sqrt{1 - \frac{\sum_{j=1}^N (F_{Ej} - F_{Rj})^2}{\sum_{j=1}^N (F_{Ej} - \bar{F}_E)^2}} = \sqrt{1 - \frac{1,34}{422,81}} = 0,9984.$$

The value means that the mathematical models (12) with a reliability of 99,84 % describe the results of the experiment, which is an excellent result and it can be concluded that the mathematical models are adequate and can be the basis for the optimization procedure.

The graph of function (12) is shown in figure 6.

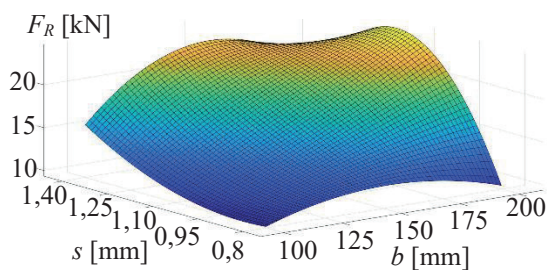


Figure 6 Mathematical model graph

According to the graph, figure 6, it is visible that the bending force constantly increases with the increase of especially small sheet thicknesses s . On the other hand, with the increase of the cross-sectional width the force increases up to a certain value (150), and then it decreases slightly. The reason is that with a large increase in the cross-sectional width the bending resistance of the sheet decreases, i.e. even with elastic deformation a greater deflection occurs.

Based on equation (12), the optimal values of the bending force can be found. Using the classical method, i.e. by finding and equating to

zero the first partial derivatives of equation (12), we obtain:

$$\frac{\partial F_R}{\partial X_1} = 0,275 \cdot X_2 - 4,41 \cdot X_1 \cdot X_2 - 9,1 \cdot X_1 \cdot X_2^2 - 6,575 \cdot X_2^2 + 7 = 0, \quad (14)$$

$$\frac{\partial F_R}{\partial X_2} = 0,275 \cdot X_1 - 13,15 \cdot X_1 \cdot X_2 - 9,1 \cdot X_1^2 \cdot X - 2,205 \cdot X_1^2 + 5,47 = 0. \quad (15)$$

By solving the system of equations (14) and (15), two realistic optimal solutions are obtained:

$$X_{11} = -0,4248 \wedge X_{21} = -1,250 \Rightarrow F_{R\min} = 11,34, \\ X_{12} = 0,4107 \wedge X_{22} = 0,7528 \Rightarrow F_{R\max} = 22,28.$$

By decoding input variables according to formulas:

$$s = 0,3X_1 + 1,1, \quad (16)$$

$$b = 50X_2 + 150, \quad (17)$$

it is obtained:

$$s_{O1} = 0,97 \wedge b_{O1} = 87,05 \Rightarrow F_{R\min} = 11,34, \\ s_{O2} = 1,22 \wedge b_{O2} = 187,64 \Rightarrow F_{R\max} = 22,28.$$

By comparing the obtained optimal solutions with the values presented in table 8, it becomes evident that the classical optimization method does not yield satisfactory results. In contrast, the Monte Carlo optimization method provides more realistic optimal solutions [11,14].

$$s_{O1} \approx 1,5 \wedge b_{O1} = 90 \Rightarrow F_{R\min} = 0,983, \\ s_{O2} \approx 1,5 \wedge b_{O2} \approx 151 \Rightarrow F_{R\max} = 26,59.$$

Based on the obtained results, the minimum bending force is achieved at a sheet metal thickness of approximately $s = 1,5$ [mm], with the target sheet metal width being around $b = 100$ [mm].

6. CONCLUSION

This paper considers approximate analytical equations for determining the bending force of a V-shaped sheet made of galvanized steel, and an experiment was conducted in real production conditions to verify them. The input parameters of the experimental design were sheet thickness

and width, as these have the most significant influence on variations in bending force. Following the experiment, it was first established that the process, at all experimental points, occurs well within the domain of plastic bending. The approximate analytical equation for bending force, which does not account for material hardening, yielded more accurate results, as confirmed by comparison with experimental data. In the bending of galvanized steel sheets, no significant hardening due to deformation was observed.

Using Design Expert 13 software, and based on the experimental design and results, a highly adequate nonlinear mathematical model was developed, with a coefficient of determination (R) exceeding 99%. Optimization was performed using both classical and Monte Carlo simulation methods, with the latter yielding more acceptable and realistic results. According to the optimization outcomes, the minimum bending force is achieved at sheet thickness values of approximately $s = 1.5$ [mm], with preferred sheet widths around $b = 100$ [mm].

The above research in this paper is the initial stage to assess which equation of approximate analytical equations for determining the bending force of a V-shaped sheet gives more accurate results. In order to be able to give a general assessment, it is necessary to repeat the research for bending sheets of other materials and for different experimental equipment. In addition to steel sheets, sheets of zinc, copper, brass, etc. should be used. Other experimental equipment includes presses from different manufacturers and other types of tools.

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