MATEMATIČKA I NUMERIČKA ANALIZA ODNOSA IZMEĐU LOSCHMIDTOVE KONSTANTE, AVOGADROVE KONSTANTE I BRZINE ZVUKA U IDEALNOM I REALNOM PLINU U STANDARDNOM STANJU

MATHEMATICAL AND NUMERICAL ANALYSIS OF THE RELATIONSHIP AMONG THE LOSCHMIDT CONSTANT, THE AVOGADRO CONSTANT, AND THE SPEED OF SOUND IN IDEAL AND REAL GASES AT STANDARD STATE

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REZIME:
U ovom radu izvedena je diferencijalna jednadžba koja daje odnos između Loschmidtove konstante, Avogadrove konstante i brzine zvuka u plinovima kao funkcije termodinamičkih svojstava plina (pritisak, temperatura i volumen). Kako bi se potvrdila valjanost izvedene jednadžbe, omjer konstanti N_L/N_A i odstupanje omjera N_L/N_A za nekoliko idealnih plinova (vodik, helij, ugljikov monoksid, dušik, kisik, ugljendioksid, metan) izračunati su pomoću jednadžbe stanja idealnog plina (Pv=RT). Izračun omjera konstanti N_L/N_A i odstupanja omjera N_L/N_A od vrijednosti za plinove u standardnom stanju (tj. temperatura 273.15 K i pritisak 101325 Pa) pokazuje dobro slaganje s objavljenim rezultatima u literaturi, s obzirom na činjenicu da literaturni podatak omjera N_L/N_A iznosi 0,04461498 u standardnom stanju. Kao drugi korak za potvrdu valjanosti izvedene jednadžbe, omjer konstanti N_L/N_A i odstupanje omjera N_L/N_A za nekoliko stvarnih plinova (vodik, helij, ugljenmonoksid, dušik, kisik, ugljendioksid, metan) izračunati su korištenjem van der Waalsove jednačine stanja stvarnog plina. U ovom slučaju rezultati proračuna također pokazuju dobro slaganje s referentnom vrijednošću.

Short original scientific paper

SUMMARY:
In this paper, a differential equation is derived relating to the relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound in gases as a function of the thermodynamic properties of the gas (pressure, temperature, and volume). To confirm the validity of the derived equation, the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for several ideal gases (hydrogen, helium, carbon monoxide, nitrogen, oxygen, carbon dioxide, methane) were calculated using the ideal-gas equations of state (Pv=RT). The calculation of the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio from the value for the gases at standard state (i.e., temperature 273.15 K and pressure 101325 Pa) is in a good agreement with the results published in literature, given the fact that the literature data of the N_L/N_A ratio is 0.04461498 at standard state. As the second step in confirming the validity of the derived equation, the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for several real gases (hydrogen, helium, carbon monoxide, nitrogen, oxygen, carbon dioxide, methane) were calculated using the real-gas Van der Waals equation of state. In this case, the calculation results are also in agreement with the reference value.

1. INTRODUCTION

In 1811, Amadeo Carlo Avogadro, an Italian professor of physics, suggested an important hypothesis *that equal volumes of all gases at the same temperature and pressure contain the same number of molecules*, or *the volume of a gas at a given pressure and temperature is proportional to the number of atoms or* *molecules regardless of the nature of the gas*, what is well known as the Avogadro's Principle (law) or Avogadro's constant (number). The greatest problem Avogadro had to resolve in his time was the confusion regarding atoms and molecules. One of his most important contributions was clearly distinguishing one from the other, stating that gases are composed of molecules, and these molecules are composed of atoms. Avogadro did not actually use the word "atom" as the words "atom" and "molecule" were used almost without difference. He believed that there were three kinds of "molecules," including an "elementary molecule" ("atom") [1]. Thus, the hypothesis was extremely visionary and its confirmation, using the kinetic theory of gases, came decades later.

The scientific community gave no great attention to his theory, so Avogadro's hypothesis was not immediately accepted. The studies by Charles Frédéric Gerhardt and Auguste Laurent on organic chemistry made it possible to demonstrate and explain the Avogadro's law, i.e. why the same quantities of molecules in a gas have the same volume. Unfortunately, related experiments with organic substances showed exceptions to the law. This was finally resolved by Stanislao Cannizzaro, as announced at the Karlsruhe Congress in 1860, four years after Avogadro's death. Cannizzaro explained that these exceptions were due to molecular dissociations at certain temperatures, and that Avogadro's law determined not only molecular masses, but atomic masses as well. In 1911, a meeting in Turin commemorated the hundredth anniversary of the publication of the Avogadro's classic 1811 paper. Thus, Avogadro's great contribution to chemistry was recognised. Rudolf Clausius, with his kinetic theory on gases, gave another confirmation of Avogadro's Law. Jacobus Henricus van't Hoff showed that Avogadro's theory was also relevant for dilute solutions. Avogadro is hailed as a founder of the atomic-molecular theory [1].

Despite the fact that Avogadro did not specify the ratio of the number of constituent particles in a sample to the amount of substance, the French physicist Jean Baptiste Perrin (1909) proposed naming the constant in honor of Avogadro. Jean B. Perrin won the Nobel Prize in Physics (1926) in a large part for his work in determinig the Avogadro's constant. Perrin's method was based on the Brouwnian motion [2].

In the years since then, several different methods (coulometry, electron mass measurement, x-ray crystal density method) have been used to estimate the magnitude of this fundamental constant. In general, accurate determinations of the Avogadro's number require the measurement of a single quantity on both the atomic and macroscopic scales, using the same unit of measurements. This became possible for the first time when physicist Robert Millikan measured the charge on an electron in 1910. The charge of a mole of electrons had been known since 1834, when Michael Faraday published his works on electrolysis. The charge was called the Faraday constant, and the best value of the constant is 96,485.3383 (3) C/mol, according to the NIST. The best estimate of the charge on an electron, based on modern experiments, is $1.60217653 \cdot 10^{-19}$ C/electron. When we divide the charge on a mole of electrons by the charge on a single electron, we obtain a value of the Avogadro's number of $6.02214154 \cdot 10^{23}$ particles/mole [3]. Since 1910, new calculations have more accurately determined the value of Farady's constant and elementary charge. Another approach to determining Avogadro's number starts with careful measurements of the density of an ultrapure sample of a material on the macroscopic scale.

In fact, Perrin originally proposed the name Avogadro number to refer to the number of molecules in one gram-molecule of oxygen, and this term is still widely used in introductory works. The change of the name to Avogadro's constant came with the introduction of the mole as a unit in the International System of Units (SI) in 1971, what recognised amount of substance as an independent dimension of measurements. Thus, Avogadro's constant was no longer a pure number, it now has a unit of measurements, the reciprocal mole (mol^{-1}) [4,5].

The Avogadro's constant is a scaling factor between macroscopic and microscopic observations of nature, and can be applied to any substance. The most significant consequence of Avogadro's law is that the gas constant has the same value for all gases. Because of its role as a scaling factor, it provides the relation between other physical constant and properties. For example, it establishes a relationship between: ■ the universal gas constant (R_u) and the Boltzmann constant (k_B): $R_u=k_B\cdot N_A$=8.314472 (15) J/mol·K; ■ the Faradey constant (F) and the elementary charge (e): $F=N_A\cdot e$=96,485.3383 (3) C/mol, and ■ the Avogadro constant within the definition of the unified atomic mass unit (u): 1 $u=M/N_A$= 1.660538782(83)·10^{-24} g.

The Loschmidt number is defined as the number of atoms in a gram atom or the number of molecules in a gram molecule. In literature, this number is frequently referred to as the Avogadro's number. But, the term Loschmidt number is reserved for the number of molecules in a cubic centimeter of a gas under standard conditions. In German language literature, it may refer to both constants by the same name, distinguished only by the units of measurement. The first actual estimation of the number of molecules in one cubic centimeter of a gas under standard conditions was made in 1865 by the Austrian physicist Johan Josef Loschmidt, a professor at the University of Vienna. The density of particles in a gas is now called the Loschmidt constant in his honor, and is approximately proportional to the Avogadro constant. The Loschmidt's method was based on the kinetic theory of gases. The kinetic theory was developed with a great success by the efforts of two scientists James Clerk Maxwell and Rudolph Clausius. Loschmidt was the first to estimate the physical size of molecules in 1865, but he did not actually calculate a value for the constant which now bears his name. The Loschmidt number is, by virtue of its definition, the same for atoms and molecules of all kinds. Though molecules may vary in size, shape and mass, the number of molecules in a gram molecule is a universal constant for all solids, liquids and gases, elements and compounds [6]. Thus, the number of molecules in a specific volume of gas is independent of the size or mass of the gas molecules. As an example, equal volumes of molecular hydrogen and nitrogen, as ideal gases, would contain the same number of molecules, as long as they are at the same temperature and pressure.

The Loschmidt constant is usually quoted at standard state of substance (i.e., temperature T_0=273,15 K and pressure p_0=1atm=1.01325 bar=101325 Pa), and recommended value is 2.6867774(47)·10^{25} particles/m^3. The pressure and temperature can be chosen freely, and must be quoted with values of the Loschmidt constant. The precision to which the Loschmidt constant is currently known is limited entirely by the uncertainty in the value of the gas constant.

The sound waves are a result of the movement of the elastic pieces of the substance environment, whether a substance is in gaseous, liquid or solid state. A change in gas density (or pressure) is transmitted in all directions with a certain speed. Number of compression and expansion of the environment, performed by the sound source by a sinuous curve in the unit of time is called *frequency* – unit is the number of cycles per second (c/s). The distance between the two maximum and minimum values in a sinuous curve represents *wavelength* – unit is the meter (m). Pressure changes represent *sound pressure* – the unit is N/m^2. Sound energy (J) per unit time (s) passing through a unit area (m^2) normal to the direction of propagation is called *the intensity of sound waves* or alternatively *the power density* – the unit is W/m^2 [7]. The quantity has a practical importance. The human ear is a very sensitive organ and at the same time it is very flexible. The lower limit of the audible intensity of sound is of the order of 10^{-12} W/m^2 and the maximum safety limit is the order of 1 W/m^2. The sound waves intensity of ordinary conversation is the order of 10^{-6} W/m^2, street traffic is 10^{-5} W/m^2 and jet plane is 10^{-2} W/m^2.

Propagation speed of sound waves depends only on the properties of the medium through which the propagation takes place. The speed of sound varies from substance to substance, for instance: sound travels most slowly in gases (the average gas speed of sound is about 330 m/s); it travels faster in liquids (the average gas speed of sound is about 1,500 m/s); and it travels at the fastest speed in solids (the average gas speed of sound is about 4,000 m/s). In an exceptionally stiff material such as diamond, sound travels at 12,000 m/s - which is around the maximum speed that sound will travel under normal conditions. In common everyday speech, speed of sound refers to the speed of sound waves in air. At 20°C, the speed of sound in air is about 343 m/s. The speed of sound in an ideal gas depends only on its temperature and composition. The speed of sound has a weak dependence on frequency and pressure in ordinary air, deviating slightly from ideal behavior.

2. MATHEMATICAL ANALYSIS

2.1. The thermodynamic speed of sound

The thermodynamic speed of sound (i.e., the speed of sound at zero frequency) in a fluid *u*, *m/s* is defined by the Laplace equation

$$u^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \tag{1a}$$

where: ρ, kg/m^3 is the density of the substance; p, N/m^2 is the pressure; s, J/kg·K is the specific entropy of the substance. Since $\rho = 1/v$, the Laplace equation has the following form, according to Ref. [8, p.127]:

$$u^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_s \tag{1b}$$

By combining the above equation with the important relationship that determines the following derivative, rarely mentioned in the literature, according to Ref. [8, p.124]:

$$\left(\frac{\partial p}{\partial v}\right)_s = \left(\frac{\partial p}{\partial v}\right)_T - \frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 \tag{2}$$

it is obtained that

$$u^2 = v^2\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 - \left(\frac{\partial p}{\partial v}\right)_T\right] \tag{3}$$

An equivalent form of Eq. (3) can be found by replacing the derivative $(\partial p/\partial T)_v$ in terms of the cyclic equation, according to Ref. [9, p.636]:

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T \tag{4}$$

so that Eq. (3) results

$$u^2 = v^2\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial v}\right)_T^2\left(\frac{\partial v}{\partial T}\right)_p^2 - \left(\frac{\partial p}{\partial v}\right)_T\right] \tag{5}$$

or

$$v = u\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial v}\right)_T^2\left(\frac{\partial v}{\partial T}\right)_p^2 - \left(\frac{\partial p}{\partial v}\right)_T\right]^{-\frac{1}{2}} \tag{5a}$$

2.2. Relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound

The Loschmidt constant is related to the Avogadro constant by relation, according to Ref. [10, p.418]:

$$N_L = \frac{p}{R_u T}N_A = \frac{\rho(p,T)}{M}N_A = \frac{1}{v(p,T)\cdot M}N_A, \quad \frac{\text{particles}}{\text{m}^3}, \tag{6}$$

where: N_L, particles/m^3 is the Loschmidt constant; p, N/m^2 is the pressure; T, K is the temperature; R_u, J/kmol·K is the universal gas constant; ρ (p, T), kg/m^3 is the density of the substance; v (p, T), m^3/kg is the specific volume of the substance; M, kg/kmol is the atomic mass of the substance, and N_A, particles /kmol is the Avogadro constant.

Combining Eq. (5a) and (6), we obtain relationship among the Loschmidt constant, the Avogadro constant, and the speed of sound, in the following form:

$$\frac{N_L}{N_A} = \frac{1}{u\cdot M}\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial v}\right)_T^2\left(\frac{\partial v}{\partial T}\right)_p^2 - \left(\frac{\partial p}{\partial v}\right)_T\right]^{\frac{1}{2}} \tag{7}$$

Since the specific volume and density are inversely proportional, that is $v=1/\rho$, the following relation for the partial derivatives is obtained:

$$\left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{\partial p}{\partial \rho}\right)_T\left(\frac{\partial \rho}{\partial v}\right)_T = -\rho^2\left(\frac{\partial p}{\partial \rho}\right)_T \tag{a}$$

and

$$\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial \rho}{\partial T}\right)_p\left(\frac{\partial v}{\partial \rho}\right)_p = -\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_p \tag{b}$$

When the previous relations are inserted in Eq. (7), relation in the following form is obtained:

$$\frac{N_L}{N_A} = \frac{1}{u\cdot M}\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial \rho}\right)_T^2\left(\frac{\partial \rho}{\partial T}\right)_p^2 + \rho^2\left(\frac{\partial p}{\partial \rho}\right)_T\right]^{1/2} \tag{8}$$

The relative deviations of the N_L/N_A ratio from the literature ratio value 0.04461498 at standard state, according to Ref. [3,6], for the gases (hydrogen, helium, carbon monoxide, carbon dioxide, nitrogen, oxygen, and methane) are calculated as follows:

$$\Delta\left(\frac{N_L}{N_A}\right)=\frac{0.04461498-\left(\frac{N_L}{N_A}\right)_{gas}}{0.04461498}100\% \tag{9}$$

3. NUMERICAL ANALYSIS

3.1. Numerical analysis of the ratio of the constants and the deviation of the N_L/N_A for the ideal gases at standard state (i.e., temperature 273.15 K and pressure 101325 Pa)

As the first step to confirm the validity of the derived equation (8), the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for several ideal gases (hydrogen, helium, carbon monoxide, argon, nitrogen, oxygen, carbon dioxide, methane) were calculated using the ideal gas equations of state ($Pv=RT$). Namely, the PvT behavior of many gases at low pressures and moderate temperatures can be modeled quite well by the ideal gas equation of state. For example, nitrogen approximates ideal behavior over a wide range of pressures up to 30 atm; argon begins to deviate after about 10 atm, and carbon dioxide is essentially independent of pressure only at extremely low pressures at a given temperature. Nevertheless, for monatomic and diatomic gases, the ideal gas eqution is usually a good approximation up to pressures of 10 to 20 atm at room temperature and above, for errors in accuracy not exceeding several percent, according to Ref. [9]. The maximum pressure at which a gas can be modeled by the ideal gas equation of state depends on the desired degree of accuracy.

If gases are ideal, the partial derivatives in equation (8) have the form:

$$\left(\frac{\partial p}{\partial \rho}\right)_T = RT, \quad \frac{\mathrm{Pa}}{\left(\mathrm{kg/m^3}\right)} \tag{10a}$$

and

$$\left(\frac{\partial \rho}{\partial T}\right)_p = -\frac{p}{RT^2}, \quad \frac{\mathrm{kg}}{\mathrm{m^3K}} \tag{10b}$$

The speed of sound in ideal gases is calculated using equation (3), and the corresponding partial derivatives of the ideal gas equation ($Pv=RT$) are:

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v}, \quad \frac{\mathrm{Pa}}{\mathrm{K}} \tag{11a}$$

and

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{v^2}, \quad \frac{\mathrm{Pa}}{\left(\mathrm{m^3/kg}\right)} \tag{11b}$$

Table 1 The data, the speed of sound and the partial derivatives of the ideal gas equation

Gas	c_v, J/kgK	R, J/kgK	v, m^3/kg	M, kg/kmol	$(\partial p/\partial T)_v$,Pa/K eq. (11a)	$(\partial p/\partial v)_T$, Pa/(m^3/kg) eq. (11b)	u, m/s; eq.(3)
	Data according to Ref. [9,11]						
H_2	10070.55	4125	11.1256	2.01588	370.7655	-9102.8686	1258.203
He	3116.176	2079	5.60284	4.002602	371.0618	-18090.0285	972.96
CO	743.170	297	0.799679	28.0101	371.399	-126860.457	337.01
N_2	742.885	296.7	0.799753	28.013	370.989	-126708.863	336.839
O_2	655.221	259.9	0.699774	31.9988	371.405	-144974.587	314.695
CO_2	632.016	189	0.505865	44.0098	373.617	-201740.802	257.963
CH_4	1656.75	518.8	1.393808	16.0428	372.217	-72944.956	429.047

Table 2 The N_L/N_A ratio calculations for the ideal gases

Gas	$(\partial p/\partial \rho)_T$,Pa/(kg/m^3) eq. (10a)	$(\partial \rho/\partial T)_p$, kg/m^{3}K eq. (10b)	N_L/N_A; eq. (8)	$(N_L/N_A)_{\text{Ref.[3,6]}}$ - $(N_L/N_A)_{\text{gas}}$	$\Delta(N_L/N_A)$, %; eq. (9)
H_2	1126743.75	-0.000329223	0.0446670296	-0.0000520486	-0.116664
He	567878.85	-0.0006532	0.04458756	0.00002742	0.061459
CO	81125.55	-0.00457254	0.0446234316	-0.0000084516	-0.0189434
N_2	81043.605	-0.004577165	0.0446247865	-0.0000098065	-0.02198028
O_2	70991.685	-0-00522526	0.04469970413	-0.00008472413	-0.1899006
CO_2	51625.35	-0.0071854	0.04501832124	-0.00040334124	-0.9040489
CH_4	141710.22	-0.002617666	0.0449278715	-0.0003128915	-0.70131489

The calculation of the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for the ideal gases at standard state (i.e., temperature 273.15 K and pressure 101,325 Pa) are in a good agreement with the results in literature, given the fact that the literature data of the N_L/N_A ratio is 0.04461498 at standard state. **Table 2** presents the absolute and relative deviations of the N_L/N_A ratio for the ideal gases, calculated using equations (8) and (9) at standard state in relation to the reference value 0.04461498 from Ref. [3,6] for the same condition. Thus, **Table 2** shows that the absolute deviation ranges from 0.0000084516 for carbon monoxide to 0.00040334124 for carbon dioxide, while the relative deviation ranges from 0.0189434% for carbon monoxide to 0.9040489% for carbon dioxide.

3.2. Numerical analysis of the ratio of the constants and the deviation of the N_L/N_A for the real gases at standard state (i.e., temperature 273.15 K and pressure 101325 Pa)

As the second step to confirm the validity of the derived equation (8), the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for several real gases (hydrogen, helium, carbon monoxide, nitrogen, oxygen, carbon dioxide, methane) were calculated using the Van der Waals equation of state, according to Ref [9].

In 1873, Van der Waals proposed an equation of state, what was the first attempt to correct the ideal gas equation, so that it would be applicable to real gases. On the bases of simple kinetic theory, particles are assumed to be point masses, and there are no intermolecular forces among particles. As the pressure increases on a gaseous system, the volume occupied by the particles may become a significant part of the total volume, and intermolecular attractive forces become important under this condition. To account for the volume occupied by the particles, Van der Waals proposed that the specific volume in the ideal gas equation to be replaced by the term $v-b$. Also, the ideal pressure was to be replaced by the term $P+a/v^2$. Thus, the Van der Waals equations of state are:

$$\left(p+\frac{a}{v_m^2}\right)\left(v_m-b\right)=R_uT \tag{12}$$

or

$$v_m^3-\left(b+\frac{R_uT}{p}\right)v_m^2+\frac{a}{p}v_m-\frac{b}{p}a=0 \tag{13}$$

or

$$\rho_m^{-3}-\left(b+\frac{R_uT}{p}\right)\rho_m^{-2}+\frac{a}{p}\rho_m^{-1}-\frac{b}{p}a=0 \tag{14}$$

where: p, bar is the pressure; ρ_m (p, T), kmol/m^3 is the molar density of the substance; v_m (p, T), m^3/kmol is the molar specific volume of the substance; a, bar·(m^3/kmol)^2 is constant; b, m^3/kmol is constant; R_u, J/kmol·K is the universal gas constant.

If gases are real, the partial derivatives in equation (8) have the form:

$$\left(\frac{\partial p}{\partial \rho}\right)_T=\frac{p\left(3p-2bp\rho_m-2R_uT\rho_m+a\rho_m^2\right)}{\rho_m^2\left(R_uT-a\rho_m+ab\rho_m^2\right)}\cdot\frac{10^5}{M},\quad \frac{\mathrm{Pa}}{\left(\mathrm{kg/m^3}\right)} \tag{15}$$

and

$$\left(\frac{\partial \rho}{\partial T}\right)_p=\frac{R_u\rho_m^2}{-3p+2bp\rho_m+2R_uT\rho_m-a\rho_m^2}\cdot M,\quad \frac{\mathrm{kg}}{\mathrm{m^3K}} \tag{16}$$

The speed of sound in real gases is calculated using equation (3), and the corresponding partial derivatives of Van der Waals equation (13) are:

$$\left(\frac{\partial p}{\partial T}\right)_v=\frac{R_u}{v_m-b},\quad \frac{\mathrm{Pa}}{\mathrm{K}} \tag{17}$$

and

$$\left(\frac{\partial p}{\partial v}\right)_T=\left[-\frac{R_uT}{\left(v_m-b\right)^2}+\frac{2a}{v_m^3}\right]\cdot M\cdot 10^5,\quad \frac{\mathrm{Pa}}{\left(\mathrm{m^3/kg}\right)} \tag{18}$$

Table 3 The calculation of the speed of sound for the real gases using Van der Waals equation

Gas	a, bar(m^3/kmol)^2	b, m^3/kmol	ρ_m, kmol/m^3	M, kg/kmol	$(\partial p/\partial T)_v$, Pa/K eq. (17)	$(\partial p/\partial v)_T$, Pa/(m^3/kg) eq. (18)	u, m/s; eq.(3)
	Data according to Ref. [9,11]						
H_2	0.247	0.0265	22.4281	2.01588	371.1795	-9114.844	1261.261
He	0.0341	0.0234	22.426	4.002602	371.162	-18111.345	973.458
CO	1.463	0.0394	22.3991	28.0101	371.874	-126516.944	336.7637
N_2	1.361	0.0385	22.4035	28.013	371.786	-126521.44265	336.7949
O_2	1.369	0.0315	22.3919	31.9988	363.8119	-144557.668	312.741
CO_2	3.643	0.0427	22.2630	44.0098	366.106	-199541.173	256.6828
CH_4	2.285	9.0427	22.3605	16.0428	372.566	-72498.121	430.465

Table 4 The N_L/N_A ratio calculations for the real gases using Van der Waals equation

Gas	$(\partial p/\partial \rho)_T$,Pa/(kg/m^3) eq. (15)	$(\partial \rho/\partial T)_p$, kg/m^{3}K eq. (16)	N_L/N_A; eq. (8)	$(N_L/N_A)_{\text{Ref.[3,6]}}$ - $(N_L/N_A)_{\text{gas}}$	$\Delta(N_L/N_A)$, %; eq. (9)
H_2	1112900.794	-0.000329119	0.0442006	0.00041438	0.92879
He	567411.299	-0.000653797	0.04455534	0.00005964	0.133677
CO	81031.722	-0.004591028	0.044674349	-0.000059369	-0.1330696
N_2	81061.654	-0.00460508	0.0447150155	-0.0001000355	-0.2242195
O_2	70864.336	-0.00524875	0.0449547837	-0.0003398037	-0.76163589
CO_2	50086.82	-0.00736251	0.0446578458	-0.0000428658	-0.096079388
CH_4	140915.435	-0.002642338	0.0447241547	-0.0001091747	-0.2447041

The calculation of the ratio of the constants N_L/N_A and the deviation of the N_L/N_A ratio for the real gases at standard state (i.e., temperature 273.15 K and pressure 101,325 Pa) are in a good agreement with the results in literature, given the fact that the literature data of the N_L/N_A ratio is 0.04461498 at standard state. **Table 4** presents the absolute and relative deviations of the N_L/N_A ratio for the real gases calculated using Vder Waals equation at standard state in relation to the reference value 0.04461498 from Ref. [3,6] for the same condition. Thus, **Table 4** shows that the absolute deviation ranges from 0.0000428658 for carbon dioxide to 0.00041438 for hidrogen, while the relative deviation ranges from 0.096079388% for carbon dioxide to 0.92879% for hidrogen.

4. CONCLUSION

- The contribution of the paper is in the derivation of equation (8), which defines the analytical relationship between the Avogadro and Loschmidt numbers as a function of the speed of sound and *pvT* (the thermodynamic properties of the gas). The Loschmidt constant is approximately proportional to the Avogadro constant, according to Ref. [6], which is also numerically confirmed for ideal and real gases (hydrogen, helium, carbon monoxide, carbon dioxide, nitrogen, oxygen, and methane) in the paper.
- The number of particles (atoms or molecules) in the same volume for all gases is equal at the same temperature and pressure. The conclusion is based on the fact that the N_L/N_A ratio is calculated using equation (8) and it is almost the same for all considered gases. Namely, the absolute differences appear on the fourth decimal place for the considered gases. This conclusion corresponds to the Avogadro's hypothesis *that equal volumes of all gases at the same temperature and pressure contain the same number of molecules*.
- The speed of sound is different in gases under the same *pvT* (the thermodynamic properties of the gas), even though in these circumstances all gases have the same number of particles in the same volumes (Avogadro's hypothesis). Thus, it is clear that the speed of sound in gases depends on the type of gas (i.e., molecular weight and heat capacity of the gas) and temperature. In other words, all gases under the same *pvT* (the thermodynamic properties of the gas) have the same number of particles (atoms or molecules) in equal volumes, but in these circumstances have different speed of sound.
- The explanation of the previous conclusion is possible by analogy with the conduction of thermal energy in gases. Sound is an energy form, as is a thermal energy. Thus, the propagation of sound in gases can be compared to the propagation of thermal energy. In the theory of heat transfer in gases and liquids, there are very successful correlations between the coefficient of heat conduction and the speed of sound in these fluids. It can be observed that the heat conduction coefficient of gases is several times higher for hydrogen and helium, compared to the coefficients of other gases, and that the same goes for the speed of sound. This is explained by the fact that hydrogen and helium have small molecular masses compared to other gases and, therefore, have a higher mean velocity of elementary particles in volume, what is directly proportional in functional relations to both the coefficient of heat conduction and the speed of sound in gas. Thus, the mean velocity of elementary particles in the volume of a gas directly and in the same way affects both the conduction of thermal energy and the propagation of sound.
- Experimental data indicate the fact that the speed of sound in gases increases as preassure increases, i.e., gas density. On the other hand, according to the kinetic theory of gases, increasing the pressure (density) of a gas

reduces the mean trajectory of a particle (atom, molecule) during the interval between two collisions. This indicates the increase of gas pressure (density) will not have as significant impact on the increase of the speed of sound in the gas as the type and temperature of the gas, because in functional relations the density and mean particle path between two collisions are proportional to both the heat conduction coefficient and the speed of sound.

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